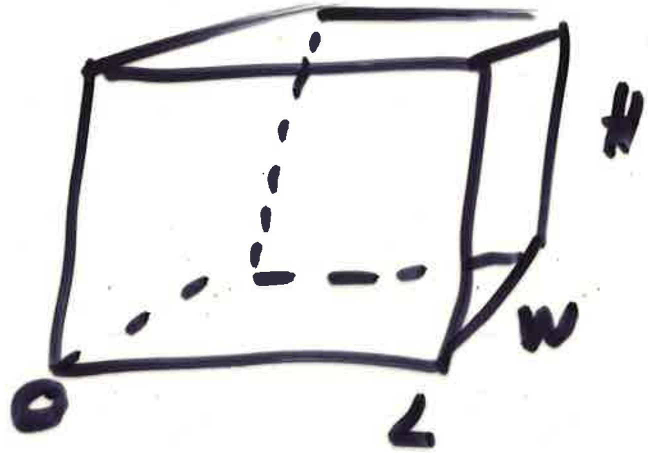
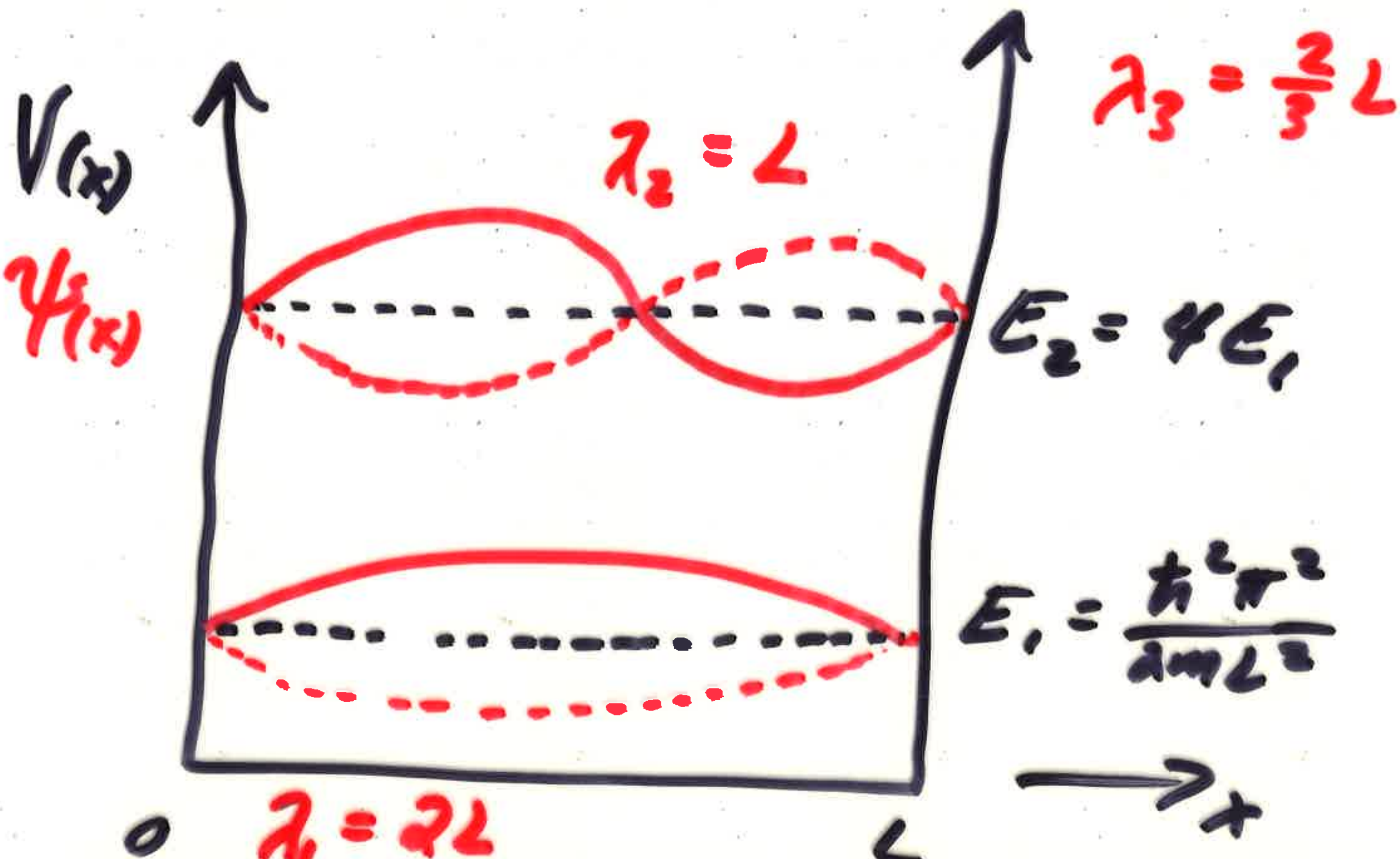
3-dimensions



$$\psi_{n_x n_y n_z}(x,y,z) = \sqrt{\frac{8}{LWH}} \sin\left(\frac{n_x \pi x}{L}\right) \sin\left(\frac{n_y \pi y}{W}\right) \cdot \sin\left(\frac{n_z \pi z}{H}\right)$$

$n_i = \{1, 2, 3, \ldots\}$

$$E_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2m}\left[\frac{n_x^2}{L^2} + \frac{n_y^2}{W^2} + \frac{n_z^2}{H^2}\right]$$



Pauli Exclusion Principle - one electron per quantum state (orbitals).

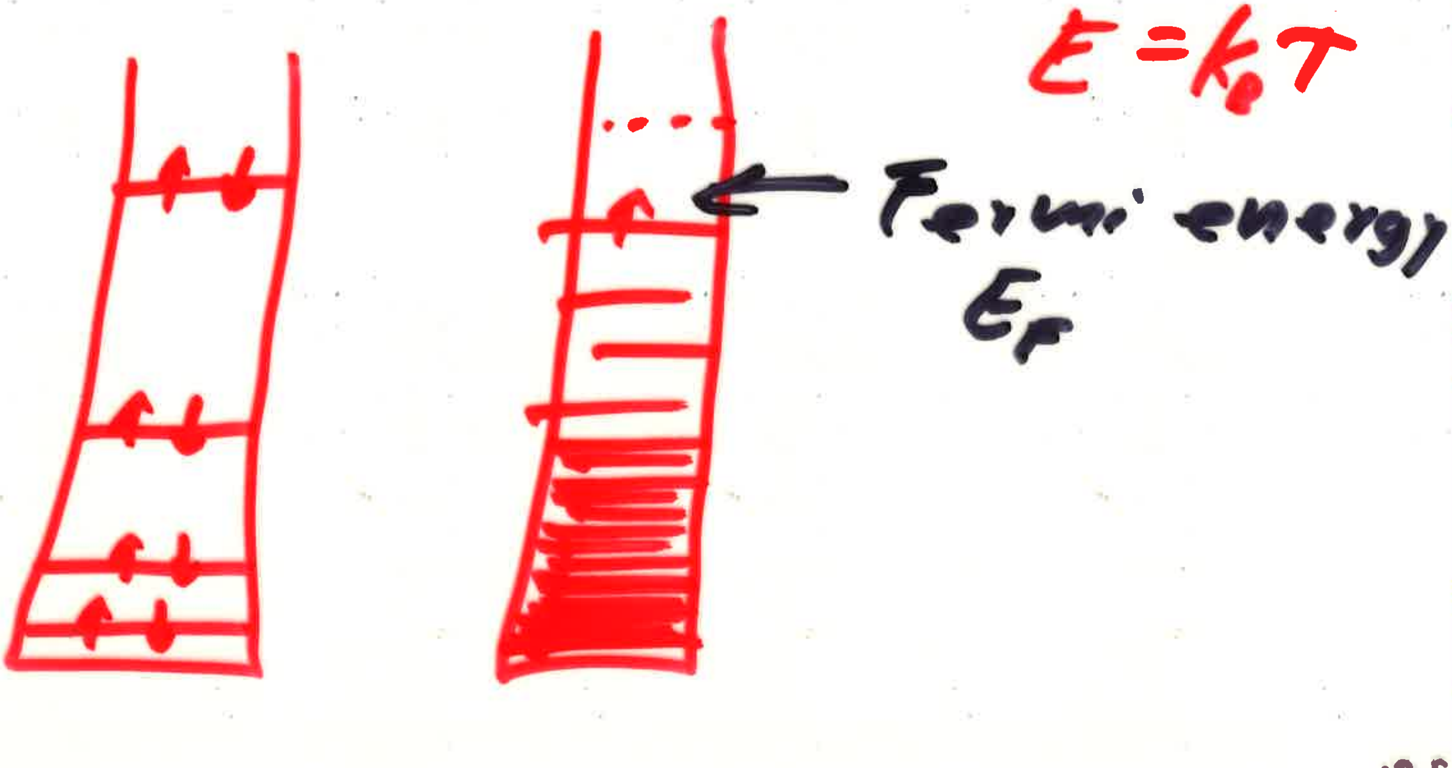
Quantum state labeled by two quantum numbers n and m_s where $m_s = \pm \frac{\hbar}{2}$ spin up / spin down

one dimension

Three dimensions

n_x, n_y, n_z, m_s

$E = k_B T$



k-space = Reciprocal Lattice

one $\vec{k}$ per $\frac{(2\pi)^3}{V}$

Density of states

two spins $\pm\frac{1}{2}\hbar$

$$D(k) \equiv \frac{dN}{dk} = \frac{2 \cdot 4\pi k^2}{\left[\frac{(2\pi)^3}{V}\right]} = \frac{V k^2}{\pi^2}$$

Cf. Lecture 24

The total number of states is

N if each atom contributes one electron e.g. Na

$$N = \int_{k=0}^{k_F} D(k)\, dk = \frac{V k_F^3}{3\pi^2}$$

k_F is the largest wavenumber "F" for Fermi

$$k_F = \left[3\pi^2 \frac{N}{V}\right]^{1/3} = \left[3\pi^2 n\right]^{1/3}$$

number density

Fermi Energy

$$E_F = \frac{\hbar^2 k_F^2}{2m} = \frac{\hbar^2}{2m}\left[3\pi^2 n\right]^{2/3}$$

e.g. Na one electron per atom

$$\frac{N}{V} = \frac{1}{V_{p\text{-cell}}} = \frac{2}{V_{\substack{\text{non-primitive}\\ \text{cube}}}} = \frac{2}{a_0^3}$$

$$E_F = \frac{\hbar^2}{2m_e}\left[3\pi^2 \frac{2}{a_0^3}\right]^{2/3} \sim 3.24\,eV$$

How are the states distributed with respect to energy

Density of states

$$D(E) \equiv \frac{dN}{dE} = ? \quad \frac{dN}{dk}\frac{dk}{dE} = D(k)\frac{dk}{dE}$$

$$\underbrace{D(E)\,dE}_{} = D(k)\,dk$$

# of states between energies E and $E + dE$.

$$k^2 = \frac{2mE}{\hbar^2} \Rightarrow k = \sqrt{\frac{2m}{\hbar^2}}\, E^{1/2}$$

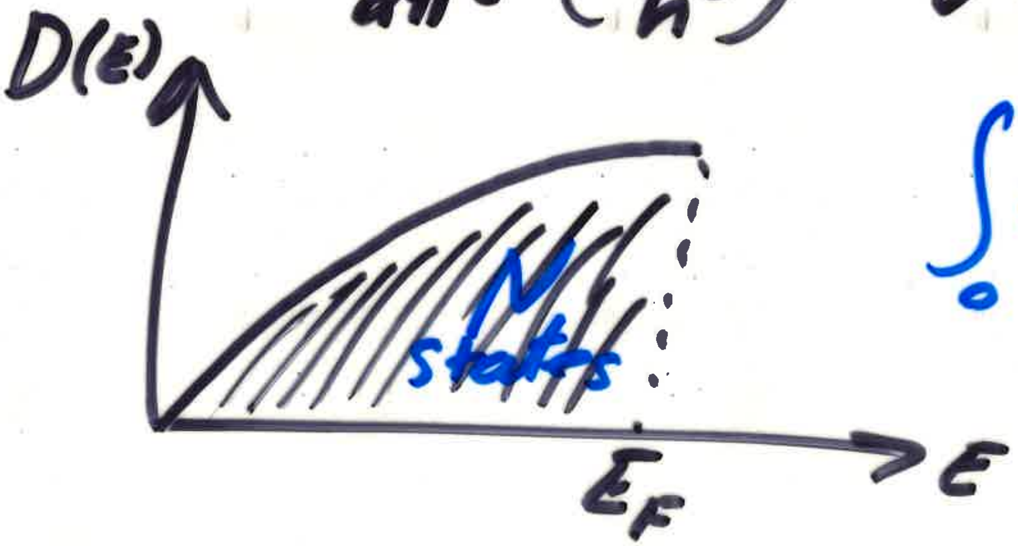
$$\frac{dk}{dE} = \sqrt{\frac{2m}{\hbar^2}}\, \frac{E^{-1/2}}{2}$$

$$D(E) = 4\pi k^2 \frac{2V}{8\pi^3} \frac{dk}{dE}$$

$$= 4\pi\left(\frac{2mE}{\hbar^2}\right)\frac{2V}{8\pi^3}\sqrt{\frac{2m}{\hbar^2}}\,\frac{1}{2\sqrt{E}}$$

$$D(E) = \frac{2V}{\hbar\, 8\pi^3}\sqrt{\frac{m}{2}}\, E^{-1/2}\, 4\pi\, \frac{2mE}{\hbar^2}$$

$$= \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E} \quad \leftarrow \checkmark$$



$$\int_0^{E_F} D(E)\, dE = N$$