<u>Bosons</u> any n, $n = 0, 1, 2, 3, \ldots$

$$\langle n \rangle = \sum_{m=0}^{\infty} m\, P(m) = 0\, P(0) + 1\, P(1) + 2P(2) + \ldots$$

$$= \frac{\displaystyle\sum_{m=0}^{\infty} m\, e^{-\frac{m(\epsilon-\mu)}{k_B T}}}{\displaystyle\sum_{p=0}^{\infty} e^{-\frac{p(\epsilon-\mu)}{k_B T}}}$$

← Gibbs weighting

← Q - Gibbs Sum
Grand Partition Function

$$Q = \sum_{p=0}^{\infty} e^{-\frac{p(\epsilon-\mu)}{k_B T}} = \sum_{p=0}^{\infty} x^p$$

where $x = e^{-\frac{(\epsilon-\mu)}{k_B T}}$

Geometric series $x < 1$

$$= \frac{1}{1-x} = \frac{1}{1 - e^{-\frac{(\epsilon-\mu)}{k_B T}}}$$

Numerator $x \dfrac{\partial Q}{\partial x} = x \displaystyle\sum_{p=0}^{\infty} p\, x^{p-1} = \sum_{\cancel{p}\,m=0}^{\infty} \cancel{p}\,m\, x^{m}$

$$= x \frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{x}{(1-x)^2}$$

$$\langle n \rangle = \frac{x \frac{dQ}{dx}}{Q} = \frac{\frac{x}{(1-x)^2}}{\left(\frac{1}{1-x}\right)} = \frac{x}{1-x}$$

$$= \frac{e^{-\frac{(\epsilon-\mu)}{k_BT}}}{1-e^{-\frac{(\epsilon-\mu)}{k_BT}}} \cdot \frac{e^{+\frac{(\epsilon-\mu)}{k_BT}}}{e^{+\frac{(\epsilon-\mu)}{k_BT}}} = \frac{1}{e^{+\frac{(\epsilon-\mu)}{k_BT}} - 1}$$

Bose-Einstein (BE) distribution

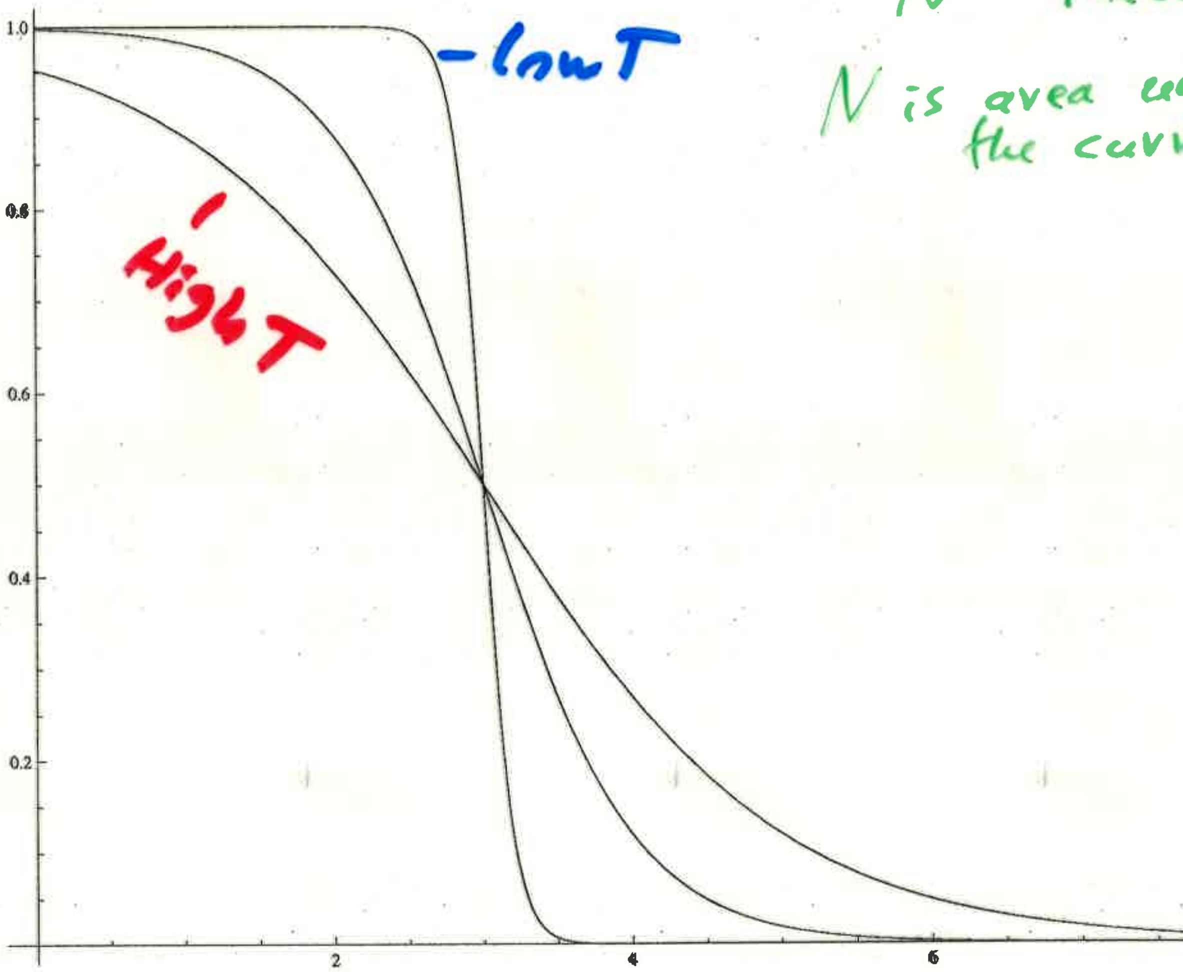
$\mu = 3$ fixed

N = fixed

N is area under the curve.

```
In[167]:= Plot[f[e], {e, 0, 10}]
```

Out[167]=



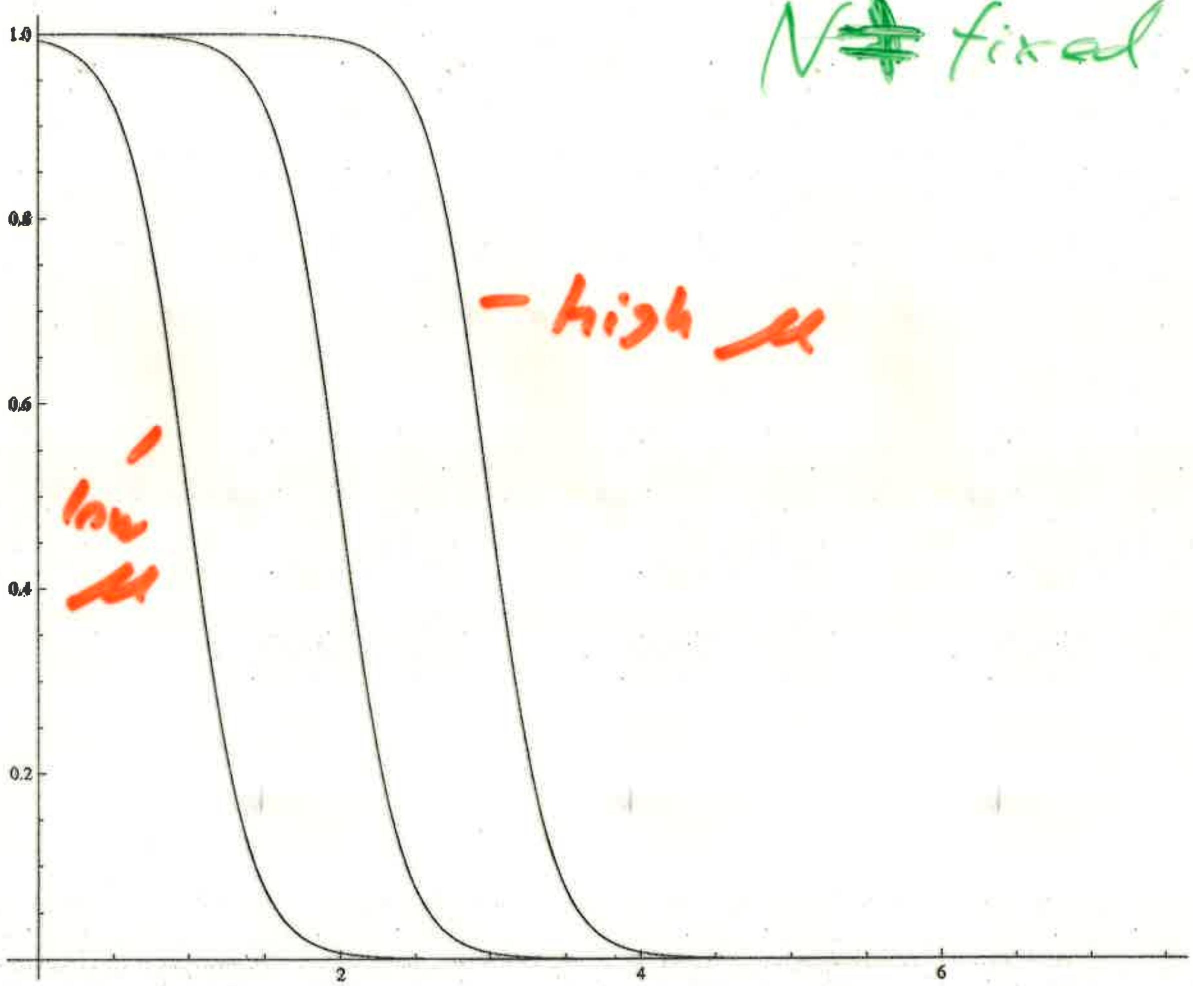

```
In[171]:= f[e_] = 1 / (Exp[(e - u) / kbt] + 1);

In[170]:= u = {1, 2, 3}; kbt = {.2, .2, .2};
```

T = fixed

N ≠ fixed

```
In[172]:= Plot[f[e], {e, 0, 10}]
```

Out[172]=




```
In[193]:= f[e_] = 1 / (Exp[(e - u) / kbt] + 1);

In[192]:= u = {5, 4.5, 4}; kbt = {.1, .5, 1};
```

N = fixed

In[184]:= Plot[f[e], {e, 0, 10}]

Out[184]=