1. Read Griffiths sections 4-3 and 4-4. Did you read all the pages?

2. Let $\hat{A}$ and $\hat{B}$ be two hermitian operators and let c be a complex number. Show your work in all of the following.

   (a) Is the sum $\hat{A} + \hat{B}$ hermitian?

   (b) Under what conditions is the product $\hat{A}\hat{B}$ hermitian?

   (c) Under what conditions is the product $c\hat{A}$ hermitian?

   (d) Is the commutator $[\hat{A}, \hat{B}]$ hermitian?

   (e) Is $\exp(i\hat{A})$ unitary? (The exponential of an operator is defined through the Taylor series.)

3. Spectrally decompose the 2×2 hermitian matrix $M = \begin{pmatrix} 4\pi & i\pi \\ -i\pi & 4\pi \end{pmatrix}$ as

$$M = \sum_{i=1}^{2} \rho_i \, \vec{u}_i \vec{u}_i^\dagger = \sum_{i=1}^{2} \rho_i \, P_i$$

   where ρ_i is the ith eigenvalue and $\vec{u}_i$ is the ith orthonormalized column eigenvector. That is,

$$M\vec{u}_i = \rho_i \vec{u}_i \qquad \text{and} \qquad \vec{u}_i^\dagger \cdot \vec{u}_j = \delta_{ij} \quad .$$

   (a) Find ρ_1, $\vec{u}_1$, ρ_2, and $\vec{u}_2$.

   (b) Find the projector matrices, P_1 and P_2.

   (c) Find $\sin(M)$ two ways:

      i. By taking advantage of the spectral decomposition: $\sin(M) = \sum_{i=1}^{2} \sin(\rho_i) \, \vec{u}_i \vec{u}_i^\dagger = \sum_{i=1}^{2} \sin(\rho_i) \, P_i$

      ii. By using the first few (?!) terms of a Taylor expansion: $\sin(M) = M - M^3/3! + M^5/5! + \ldots$

      iii. Comment on the convergence of the series.

4. Consider the three-dimensional isotropic quantum simple harmonic oscillator.

   (a) Write the potential energy in Cartesian coordinates $V(x, y, z)$.

   (b) Write the potential energy in spherical polar coordinates $V(r, \theta, \phi)$.

   (c) Write the Schrödinger equation in Cartesian coordinates in detail.

   (d) Write the ground state wavefunction in Cartesian and in spherical polar coordinates.

   (e) What is the ground state energy?

   (f) Find the four lowest unique energy eigenvalues and their degeneracies.

Bonus:

1. Find the formula for the degeneracy of the nth energy level of the three-dimensional quantum SHO.
2. Find $\sqrt{M}$, where M is given above.