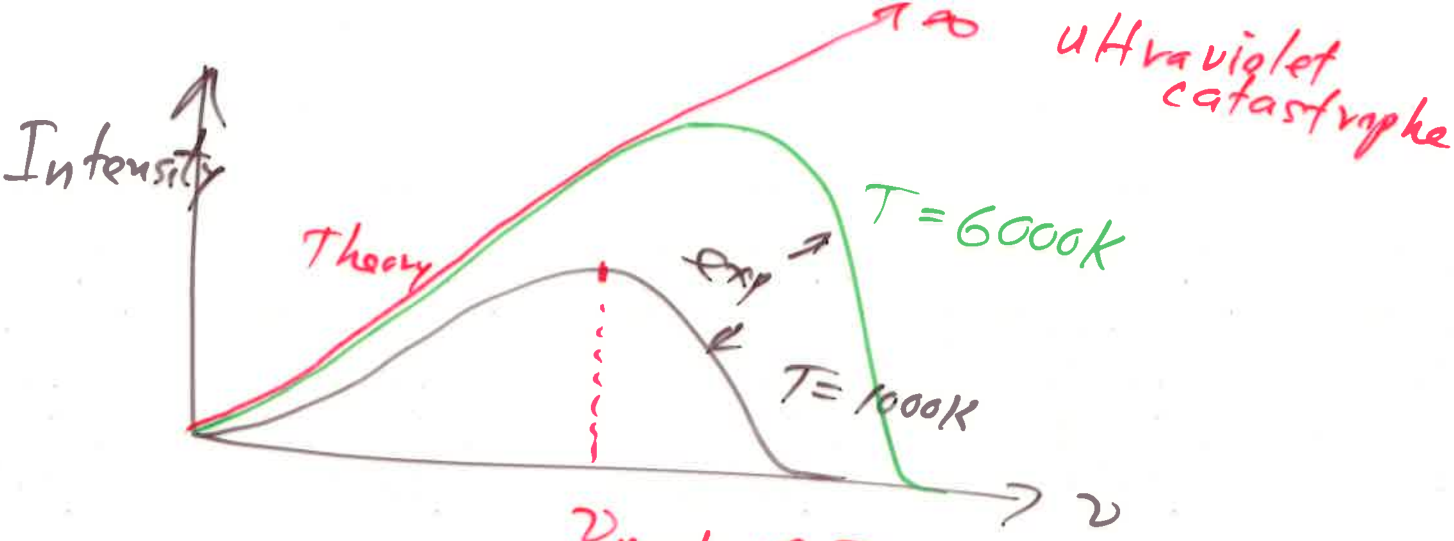
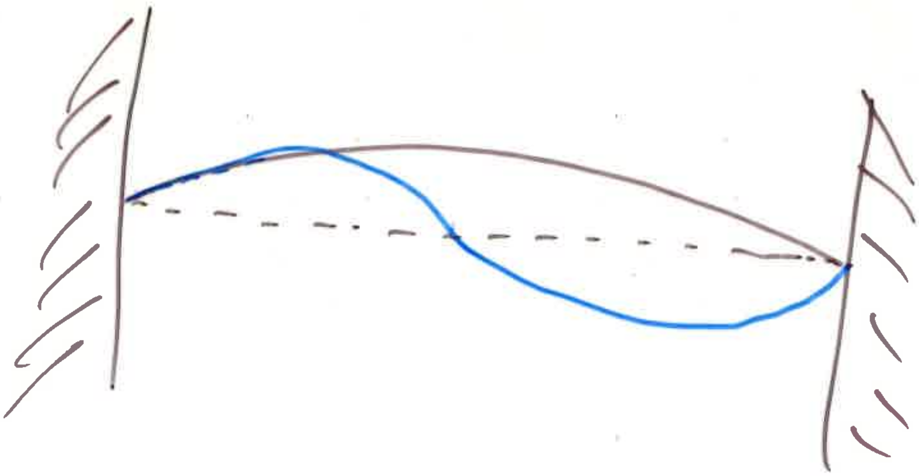


$\nu_{peak} \propto T$

$E_{tot} \propto T^4$



Classical
frequency
$f = \nu$ fixed
Any energy I want

Quantum $E = hf = h\nu$

$E = 2hf = h\nu$

$\vdots$

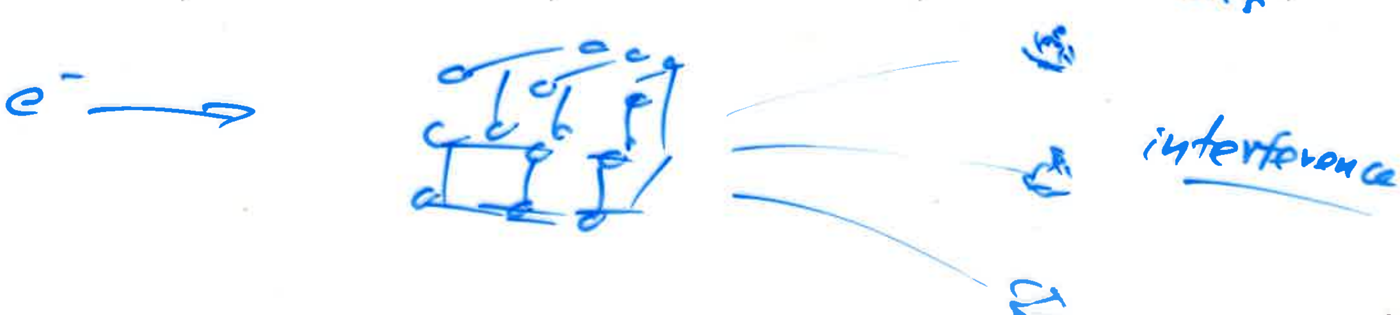
$E = nh\nu$

$n \in$ Integer

$\uparrow$ Quantum number

= number of photons

Louis deBroglie



Schrödinger (Erwin) = Schroedinger

found wave equation for matter waves

Math tools

vectors $\vec{V} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} = |V\rangle$

← column vector (n-tuple)

↑ ket

Dirac bracket notation

dagger = hermitian conjugate

$h \to \infty$

$\vec{V}^{\dagger} = \vec{V}^{T*} = (v_1^*, v_2^*, \ldots v_n^*) = \langle V|$

← complex conjugate

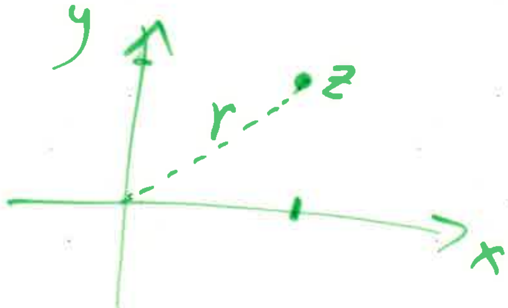
↑ transpose

↑ bra

Complex numbers

Cartesian: $z = x + iy$ $\qquad x, y \in \text{Real}$

Polar: $z = re^{i\theta}$ $\qquad i = \sqrt{-1}$



$r = \sqrt{x^2+y^2} \in \text{Real}$

$\theta = \arctan\left(\frac{y}{x}\right) \in \text{Real}$

↑ phase angle

$z^* = x - iy$ $\qquad i \to -i$

$z^* = re^{-i\theta}$

Normed vector space

$\underbrace{\text{Inner product}}_{\text{dot product}}$ space

$$\vec{A} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}, \quad \vec{B} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

$$\vec{A}^{*} \cdot \vec{B} \longrightarrow \underset{\uparrow \text{row}}{A^{T*}} \underset{\uparrow \text{column}}{B} = A^{+}B = \langle A | B \rangle$$

$$\underset{1 \times n}{(\ldots)} \underset{n \times 1}{\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}} = \underset{1 \times 1}{(\cdot)}$$

$\uparrow$ complex scalar

Norm of a vector "Length" of $|A\rangle$

$$\sqrt{\vec{A}^{*} \cdot \vec{A}} = \sqrt{\langle A | A \rangle}$$

$\vec{A}^{*} \cdot \vec{A}$ is real

$$\vec{A} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$ complex

$$a_1 = x_1 + i y_1$$
$$a_2 = x_2 + i y_2$$
$$\vdots$$

$$(-i) \cdot (i) = +1$$

$$\vec{A}^{*} \cdot \vec{A} = \underbrace{(x_1 - i y_1)}_{a_1^{*}} \underbrace{(x_1 + i y_1)}_{a_1} + \underbrace{(x_2 - i y_2)}_{a_2^{*}} \underbrace{(x_2 + i y_2)}_{a_2} + \ldots$$

$$= x_1^2 + y_1^2 + x_2^2 + y_2^2 + \ldots x_n^2 + y_n^2$$

Every finite dimensional normed vector space is automatically a Hilbert space.

We need an infinite dimensional space for Quantum Mechanics.

The "vectors" in the Hilbert space must be square integrable.

"vectors" are complex valued functions

$|f\rangle = f(x)$ is in the Hilbert space

if ① there is "dot product" = inner product

$$\langle g|f\rangle = \int_a^b g^*(x)\, f(x)\, dx = \sum_{i=1}^{n} g_i^* f_i$$

②

$$\langle f|f\rangle = \int_a^b f^*(x)\, f(x)\, dx < \infty$$

Mathematicians call this L_2

$$\langle f|g\rangle = \int_a^b g(x)\, f^*(x)\, dx \neq \langle g|f\rangle$$

$$\langle f|g\rangle = \langle g|f\rangle^*$$