Start with $|\Psi\rangle = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle$

Measure $\hat{H} \rightarrow$ get, say, E_2

now $|\Psi\rangle = |2\rangle$

$\hat{H}|2\rangle = E_2|2\rangle$ still state $|2\rangle$

Now measure $\hat{x}$

$|2\rangle$ is <u>not</u> an eigenstate of operator x

$$|2\rangle = \sum_{k=1}^{\infty} b_k |x_k\rangle \qquad b_k = \underline{\langle x_k}|2\rangle$$

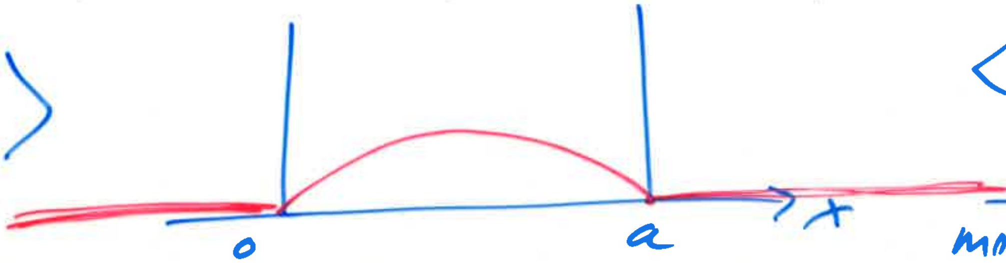
$x|2\rangle \Rightarrow$ wavefunction will collapse to one of the $|x_k\rangle$s with probability $(b_k)^2$

$$|x_{23}\rangle = \sum_{n=1}^{\infty} c_n |n\rangle$$

$\hat{A}, \hat{B}$ commute iff $[\hat{A},\hat{B}] = 0$

$$\hat{A}\hat{B}|\Psi\rangle - \hat{B}\hat{A}|\Psi\rangle = 0$$

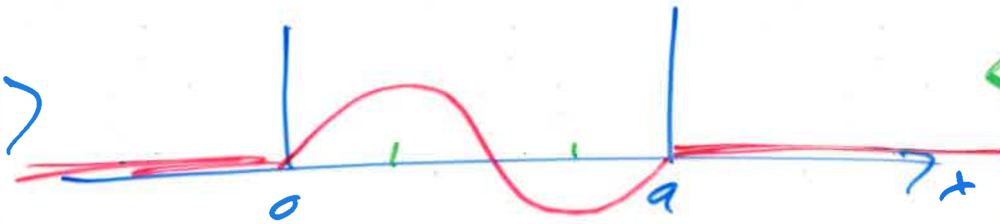
$|1\rangle$



$\langle 1|\hat{x}|1\rangle = ?$ $\frac{a}{2}$

most likely x? $\frac{a}{2}$

$|2\rangle$



$\langle 2|\hat{x}|2\rangle = ? = \frac{a}{2}$

zero chance of measuring $\frac{a}{2}$

most likely x values $\frac{a}{4}$, $\frac{3a}{4}$

$$\langle 1|\hat{p}|1\rangle = 0 = \langle 2|\hat{p}|2\rangle$$

If you prepare a state

$$|\Psi\rangle = \sum_{n=1}^{\infty} c_n |n\rangle$$

energy eigenstates

Probability of measuring E_{13}? $|c_{13}|^2$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle$$

Prob of getting $E_1 = |c_1|^2 = \frac{1}{2}$

$E_2 = |c_2|^2 = \frac{1}{2}$

We just finished a particle in 1-dimensional box

$$V(x) = \begin{cases} 0, & 0 \le x \le a \\ \infty, & \text{elsewhere} \end{cases}$$

We found energy eigenstates that were <u>complete</u>

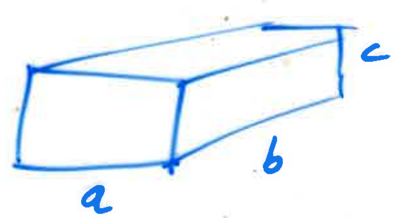
complete = can expand <u>any</u> wavefunction in the energy eigenstate basis $|1\rangle, |2\rangle, \ldots |n\rangle \ldots$

$$|n\rangle = \Psi_n(x,t) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \exp\left[\frac{-it}{\hbar}\underbrace{\left(\frac{n^2\pi^2\hbar^2}{2ma^2}\right)}_{E_n}\right]$$

$$\varphi(x,t) = \sum_{n=1}^{\infty} c_n |n\rangle \quad \text{with} \quad \sum_{n=1}^{\infty} |c_n|^2 = 1$$

For free, we can solve the particle in a 3-dimensional box

$$V(x,y,z) = \begin{cases} 0, & 0 \le x \le a \;\&\; 0 \le y \le b \;\&\; 0 \le z \le c \\ \infty, & \text{elsewhere} \end{cases}$$



Ansatz: $\Psi(x,y,z,t) = f_1(x) \cdot f_2(y) \cdot f_3(z) \cdot g(t)$

S.E. $i\hbar \frac{\partial}{\partial t} \Psi_{(x,y,z,t)} = \frac{-\hbar^2}{2m} \nabla^2 \Psi_{(x,y,z,t)} + V_{(x,y,z)} \Psi_{(x,y,z,t)}$

Second-order linear (in Ψ) homogeneous partial D.E.

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Still no time dependence.

Separation of variables $\Longrightarrow$ 4 O.DEs

$f_1(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$ $\quad \underline{n} = 1, 2, 3, \ldots$

$f_2(y) = \sqrt{\frac{2}{b}} \sin\left(\frac{\ell \pi y}{b}\right)$ $\quad \underline{\ell} = 1, 2, 3 \ldots$

$f_3(z) = \sqrt{\frac{2}{c}} \sin\left(\frac{q\pi z}{c}\right)$ $\quad \underline{q} = 1, 2, 3 \ldots$

quantum number $\Longleftrightarrow$ mode numbers

$g(t) = 1 \exp\left(\frac{-it}{\hbar} E\right)$

separation constant = energy

$$E_{n\ell q} = \frac{\left(\frac{n^2}{a^2} + \frac{\ell^2}{b^2} + \frac{q^2}{c^2}\right)\pi^2 \hbar^2}{2m}$$

$$\Psi_{(x,y,z,t)\,n\ell q} = \sqrt{\frac{8}{abc}} \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{\ell\pi y}{b}\right) \sin\left(\frac{q\pi z}{c}\right) \exp\left[\frac{-it}{\hbar} \frac{\left(\frac{n^2}{a^2} + \frac{\ell^2}{b^2} + \frac{q^2}{c^2}\right)\pi^2 \hbar^2}{2m}\right]$$

$$= |n, \ell, q\rangle$$

These 3-dimensional eigenkets are complete

$$\varphi_{(x,y,z)} = \sum_{n=1}^{\infty} \sum_{l=1}^{\infty} \sum_{q=1}^{\infty} c_{nlq} \, |n,l,q\rangle \Big|_{t=0}$$

complex numbers

basic vectors
" states
" kets
energy eigenstates

Back to 1 dimension

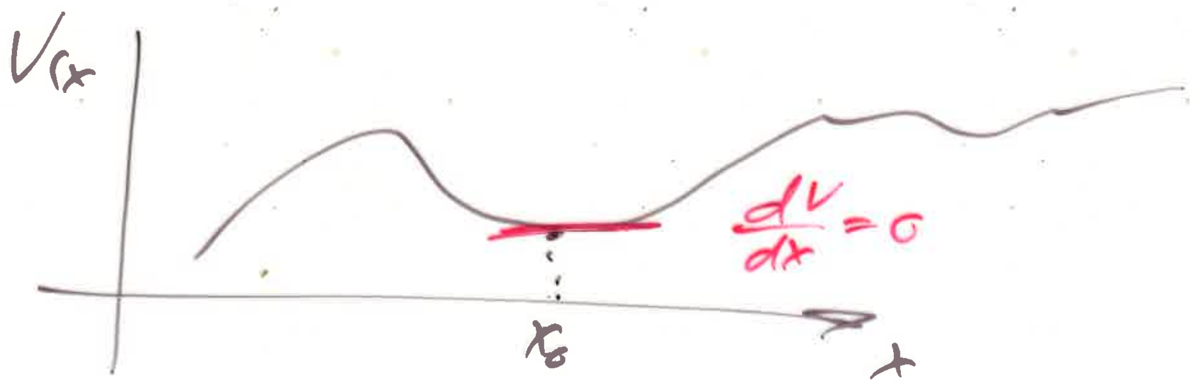
The Harmonic Oscillator

$$V(x) = \frac{1}{2} k x^2 + \cancel{\text{constant}}^{\,0} \longrightarrow \frac{1}{2} k (x - x_0)^2$$

looks like a Hooke's law spring

Why? A) Solvable (like particle in box)

B) approximation to any potential that has a minimum.



Taylor Expansion of $V(x)$ near $x = x_0$:

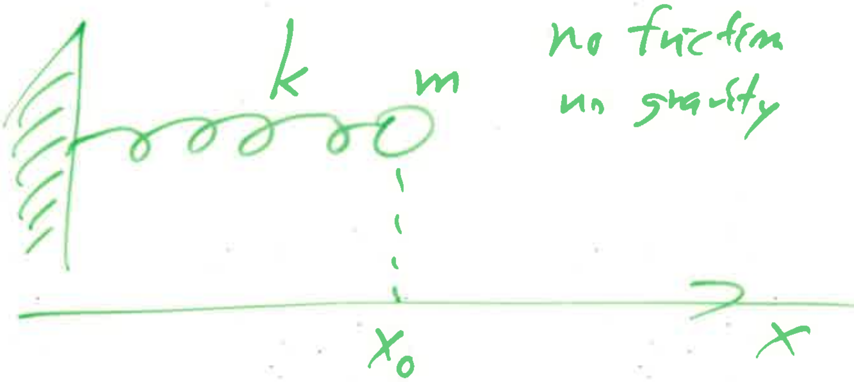
$$V(x) = V(x_0) + \cancel{(x-x_0)\left.\frac{dV(x)}{dx}\right|_{x=x_0}}^{\,0} + \frac{1}{2!}(x-x_0)^2 \boxed{\left.\frac{d^2V(x)}{dx^2}\right|_{x=x_0}} + \ldots$$

constant set to zero   minimum k

$$V(x) = \frac{1}{2}k(x-x_0)^2 + \ldots$$

get rid of k

Classical Mechanics   Mass on a Spring



no friction
no gravity

$$F = ma$$

$$-kx = m\frac{d^2x(t)}{dt^2}$$

$$\ddot{x}(t) = -\frac{k}{m}x(t)$$

$$x(t) = A\sin(\omega t) + B\cos(\omega t)$$

$$\dot{x}(t) = \omega A\cos(\omega t) - \omega B\sin(\omega t)$$

$$\ddot{x}(t) = -\omega^2\left[A\sin(\omega t) + B\cos(\omega t)\right] = -\omega^2 x(t)$$

$$\frac{k}{m} = \omega^2 \implies k = m\omega^2$$