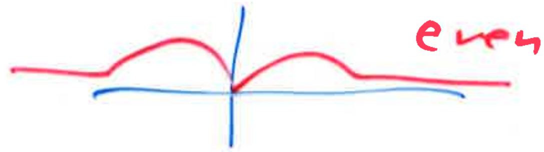
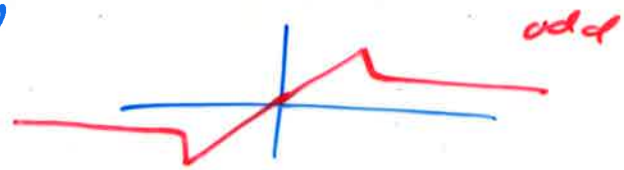
Parity: even or odd?

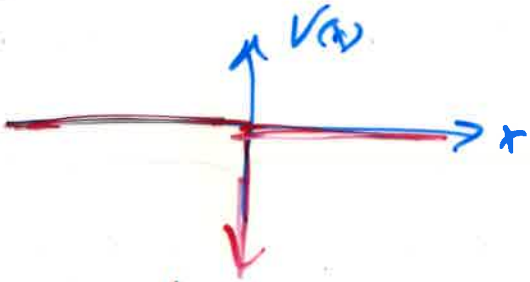
$f(x) = + f(-x)$ e.g. cosine even

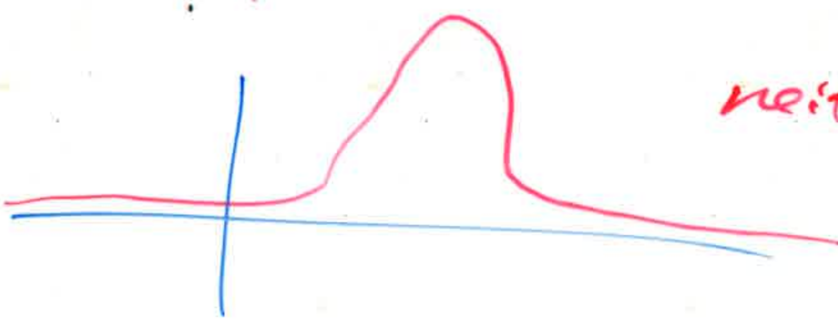


$f(x) = - f(-x)$ odd

e.g. sine







neither even nor odd.

OR

$\underrightarrow{A = 0}$ no incoming particles from left

$\underleftarrow{B e^{-ikx}}$ transmitted

$\underrightarrow{F e^{+ikx}}$ reflected

$\underleftarrow{G e^{-ikx}}$ incoming from right

2 equations, 3 unknowns.

$$\underrightarrow{Ae^{ikx}}$$

A can be anything
(not normalizable)
can divide through by A

Continuity of $f(x)$ at $x=0$

$$A + B = F \Rightarrow 1 + \left(\frac{B}{A}\right) = \left(\frac{F}{A}\right) \Rightarrow \boxed{1 + \tilde{B} = \tilde{F}}$$

dis-Continuity of $f'(x)$ at $x=0$

$$ik(F - A + B) = -\frac{2m}{\hbar^2}\alpha(F)$$

$$\boxed{ik(\tilde{F} - 1 + \tilde{B}) = \frac{-2m\alpha}{\hbar^2}\tilde{F}}$$

2 equations, 2 unknowns: $\tilde{B}, \tilde{F}$.

$$\tilde{B} = \frac{i\frac{m\alpha}{\hbar^2 k}}{1 - \frac{im\alpha}{\hbar^2 k}}, \quad \tilde{F} = \frac{1}{1 - \frac{im\alpha}{\hbar^2 k}}$$

Reflection Coefficient

$$R \equiv |\tilde{B}|^2 = (\tilde{B}^*)\tilde{B} = \frac{\left(\frac{m\alpha}{\hbar^2 k}\right)^2}{1 + \left(\frac{m\alpha}{\hbar^2 k}\right)^2}$$

real, positive semidefinite (non-negative), < 1

R = probability of the wave being reflected.

Transmission Coefficient

$$T \equiv |\tilde{F}|^2 = (\tilde{F}^*)\tilde{F} = \frac{1}{1 + \left(\frac{m\alpha}{\hbar^2 k}\right)^2}$$

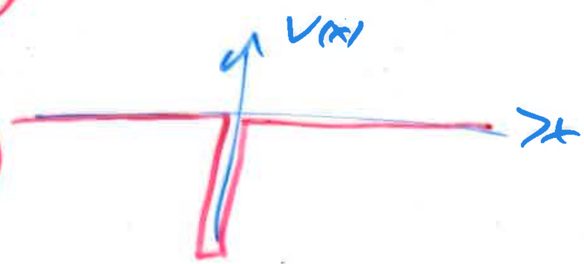
real, non-negative, < 1

$$R + T = 1 \qquad |\tilde{B}|^2 + |\tilde{F}|^2 = 1$$

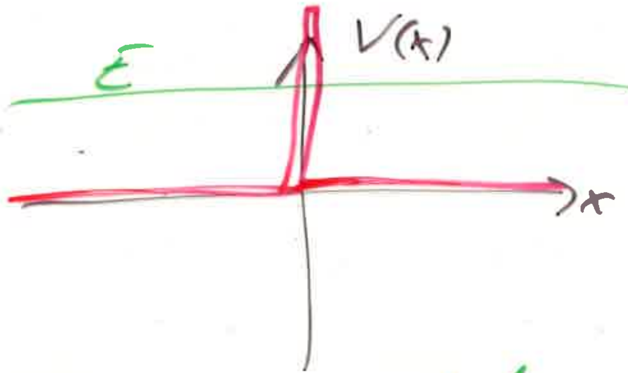
$$k = \sqrt{\frac{2mE}{\hbar^2}}, \quad E > 0$$

$$R = \frac{1}{1 + \left(\frac{2\hbar^2 E}{m\alpha^2}\right)}$$

$$T = \frac{1}{1 + \left(\frac{m\alpha^2}{2\hbar^2 E}\right)}$$



Delta Function Barrier



$V(x) = \alpha\,\delta(x)$

no bound states $E<0$

All scattering states, any E is allowed
→ E not quantized.

$$f_L(x) = \underset{\rightarrow}{A e^{ikx}} + \underset{\leftarrow}{B e^{-ikx}}$$

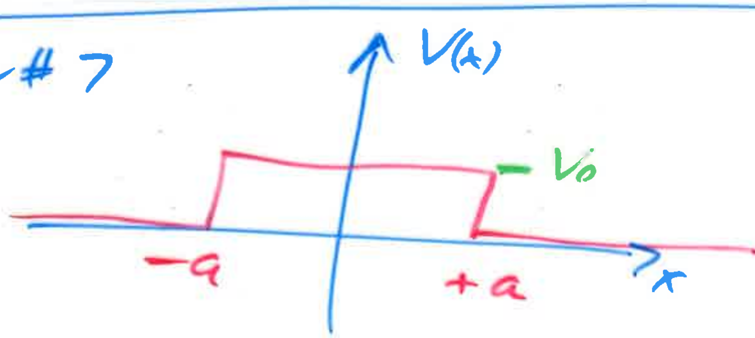
$$f_R(x) = \underset{\rightarrow}{F e^{ikx}} + \cancel{G e^{-ikx}}^{\,0}$$

$\hat{B}, \hat{F}$ same as for the well.

R and T same as for the well!

$T \neq 0 \Rightarrow$ is called "tunneling"

HW#7



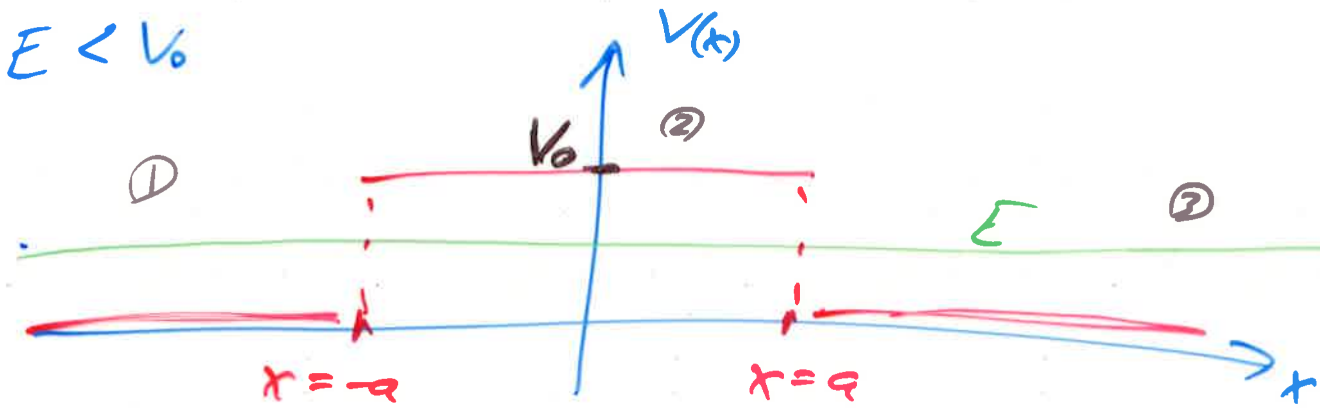
$E > 0$

$0 < E < V_0$

$E = V_0$

$E > V_0$

$E < V_0$



$$f_1(x) = \underbrace{A e^{ikx}}_{\text{incoming}} + \underbrace{B e^{-ikx}}_{\text{reflected}} \qquad k \equiv \sqrt{\frac{2mE}{\hbar^2}}$$

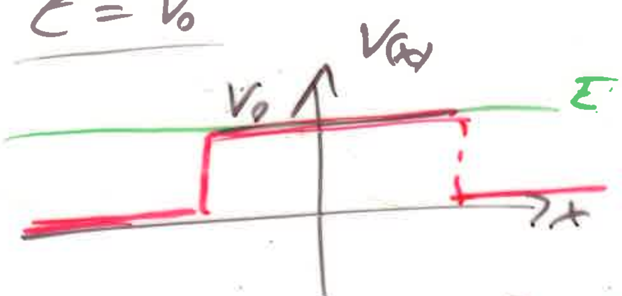
$$f_2(x) = C e^{+\gamma x} \quad D e^{-\gamma x} \qquad \gamma \equiv \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

$$f_3(x) = \underbrace{F e^{ikx}}_{\text{transmitted}} + \cancel{G e^{-ikx}} \rightarrow 0$$

no incoming from right.

tunneled through barrier.

$E = V_0$

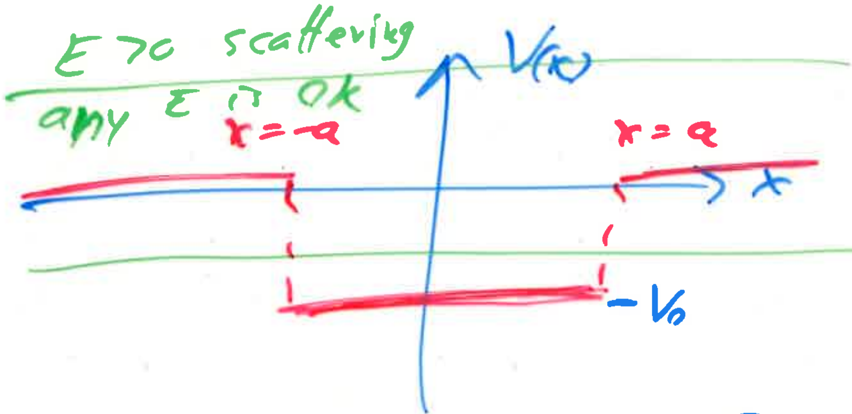


② S.E. $\quad -\frac{\hbar^2}{2m} f_2''(x) + V_0 f_2(x) = E f_2(x)$

Solve for $f_2(x)$.

Finite Square Well

$$V(x) = \begin{cases} -V_0, & -a \le x < a \\ 0, & \text{elsewhere} \end{cases}$$



E < 0 ⟹ bound states
E is quantized

$V(x)$ is even ⟹ $f(x)$ will be even or odd

($f(x)$ will have a definite parity)

an infinite ($\aleph_1$) number of scattering states.

3 bound states in this example.

Why do the bound states' parity alternate even, odd, even, odd, ... ?