CSCO - complete set of commuting observables

$$\{\hat{H}, \hat{L}^2, \hat{L}_z, \hat{S}^2, \hat{S}_z\}$$

$$\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x = x\frac{\hbar}{i}\frac{\partial}{\partial y} - y\frac{\hbar}{i}\frac{\partial}{\partial x}$$

$r\sin\theta\cos\varphi$ $\qquad$ $r\sin\theta\sin\varphi$

$\hat{L}_z|\Psi\rangle$

$\{x\ y\ z\} \rightarrow \{r, \theta, \varphi\}$

$$\hat{L}_z = \begin{pmatrix} \hbar & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar \end{pmatrix}$$

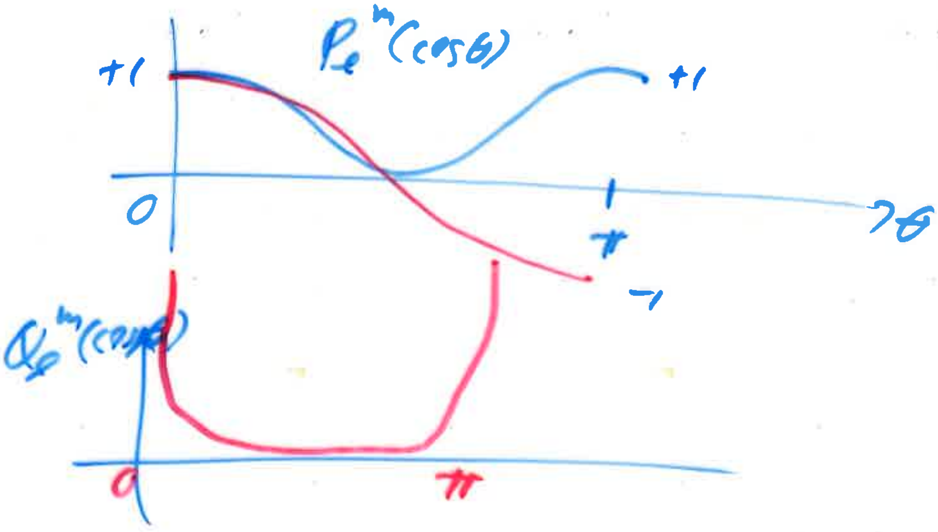
$$\hat{L}_z = \frac{\hbar}{i}\frac{\partial}{\partial\varphi} \qquad Y_1^{+1}(\theta,\varphi) = |1,1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\hat{L}_z|Y_\ell^m(\theta,\varphi)\rangle = m\hbar\,|Y_\ell^m(\theta,\varphi)\rangle$$

If ℓ = integer $\Rightarrow P_\ell^m(\theta)$ is a polynomial.

$P_\ell^m(\theta)$ Legendre function of the 1^{st} kind

$Q_\ell^m(\theta)$ " " 2^{nd} "



Commutator transitive?

$$[\hat{L}_x, \hat{L}^2] = 0 \quad [\hat{L}_y, \hat{L}^2] = 0, \quad [\hat{L}_x, \hat{L}_y] \neq 0$$

$$[\hat{L}_z, \hat{x}] = ? = [(\hat{x}\hat{p}_y - \hat{y}\hat{p}_x), \hat{x}]$$

$$= \quad 0 \quad - \hat{y}[\hat{p}_x, \hat{x}] = -\hat{y}(-i\hbar)$$

$$= + i\hbar \hat{y}$$

$$[\hat{x}, \hat{p}_x] = i\hbar$$

$$[\hat{L}_z, \hat{p}_x] = i\hbar \hat{p}_y$$

$$[\hat{L}_z, \hat{r}^2] = [\hat{L}_z, \hat{x}^2 + \hat{y}^2 + \hat{z}^2] = 0$$

$\frac{\hbar}{i}\frac{d}{d\varphi}$ no φ y x

$$[\hat{L}_k, \hat{r}^2] = 0$$

$$[\hat{L}_z, \hat{p}^2] = [\hat{L}_z, \hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2] = 0$$

x, y

$$[\hat{L}_k, \hat{p}^2] = 0$$

$$[\hat{L}^2, \hat{r}^2] = 0, \quad [\hat{L}^2, \hat{p}^2] = 0$$

$$\hat{H} = \hat{H}(\hat{p}^2, \hat{r}^2) = \frac{\hat{p}^2}{2m} + V(\hat{r}) = \frac{\hat{p}^2}{2m} + V(\sqrt{\hat{r}^2})$$

Spin-$\frac{1}{2}$ $s = l = \frac{1}{2}$ $L^2 = l(l+1)\hbar^2$

Kets: up $= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ down $= \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\hat{S}_z = \frac{1}{2}\begin{pmatrix} \hbar & 0 \\ 0 & -\hbar \end{pmatrix}$$

$$|\hat{\vec{S}}| = \sqrt{\hat{S}^2} = \sqrt{s(s+1)\hbar^2} = \frac{\sqrt{3}}{2}\hbar$$

$$\hat{S}_z|\uparrow\rangle = \frac{1}{2}\begin{pmatrix} \hbar & 0 \\ 0 & -\hbar \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2}\hbar\begin{pmatrix} 1 \\ 0 \end{pmatrix} = m_s\hbar\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\hat{S}_z|\downarrow\rangle = \frac{1}{2}\begin{pmatrix} \hbar & 0 \\ 0 & -\hbar \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\frac{1}{2}\hbar\begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

Spin-$\frac{3}{2}$ $s = \frac{3}{2}$ $m_s = \left(-\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}\right)$

Spin-2 $s = 2$ $m_s = (-2, -1, 0, 1, 2)$

$$[\hat{S}_i, \hat{S}_j] = i\hbar\,\epsilon_{ijk}\hat{S}_k$$

Pauli Matrices $\{\sigma_k\}$

$$\hat{S}_z = \frac{\hbar}{2}\sigma_z, \quad \hat{S}_x = \frac{\hbar}{2}\sigma_x, \quad \hat{S}_y = \frac{\hbar}{2}\sigma_y$$

$$\sigma_z = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\mathbb{I}_{2\times 2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$