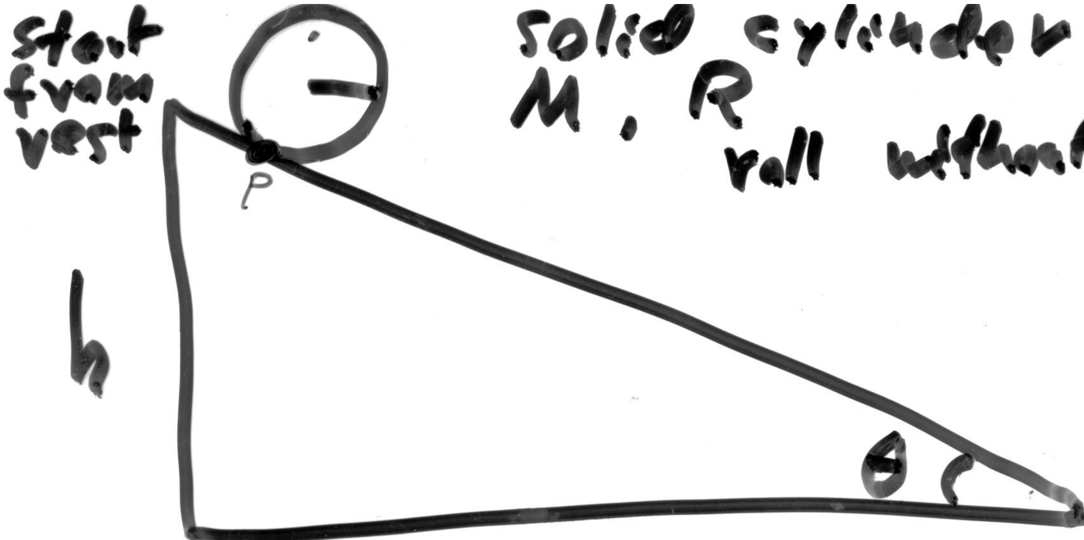


solid cylinder

M, R

roll without slipping

speed v at bottom?

acceleration?

frictional force?

normal force?

Do this in as many ways as you can.

① Newton's 2^{nd} Law

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

environment — everything in Universe <u>except</u> the particle

particle's kinematics

3-dimensional equation (in general)

If mass m = constant $\Longrightarrow \sum \vec{F} = m\vec{a} = m\frac{d^2\vec{x}(t)}{dt^2}$

$\vec{x} \equiv \vec{r}$

$= m\ddot{\vec{x}}(t)$

IF you know all the forces, you can get $\vec{a}(t) \rightarrow \vec{v}(t) \rightarrow \vec{x}(t)$

<u>Differential Equation</u>

usually <u>2^{nd} order</u> (unless $\vec{F}$ depends $\dddot{x}(t)$)

$\Rightarrow$ 2 constants of integration which will be fixed by initial conditions (boundary conditions)

e.g. $x(0) = a$, $\dot{x}(0) = b$ <u>or</u> $x(0) = a$, $x(t_0) = c$

<u>Linear</u> (in $x(t)$) every term has zero or one derivatives of $x(t)$ including zeroth derivative which is $x(t)$ itself

e.g. SHO: $\ddot{x}(t) + \omega_0^2 x(t) = 0$ Linear in x

Forced Oscillator with Damping $\ddot{x}(t) + 2\beta\dot{x}(t) + \omega_0^2 x(t) = \frac{F_0}{m}\cos(\omega t)$

Non-linear equations

$$\left[\dot{x}(t)\right]^2 + x(t) = 7$$

Simple pendulum

$$\ddot{x}(t) + \frac{g}{\ell}\sin[x(t)] = 0$$

options because hard to solve analytically.

① Linearize – e.g. small angle approximation

If x small, then $\sin[x] = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$$\approx x$$

$$\Longrightarrow \ddot{x}(t) + \frac{g}{\ell}x(t) = 0$$

② Numerically on a computer.

Stanislaw Ulam: "The study of non-linear physics is like the study of non-elephant biology."

Non-homogeneous → ($\sum F$) has no $x(t)$ or derivatives.

Angular form of Newton's 2nd Law

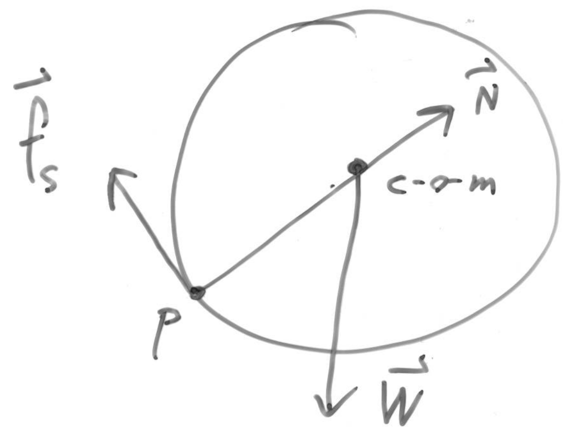
$$\sum \vec{F} = m\ddot{x}(t)$$ mass = inertia

$$\rightarrow \sum \vec{\tau}_o = I_o \vec{\alpha}$$

moment of inertia

Origin O can be $\begin{cases} \text{P - a point that does \underline{not} accelerate.} \\ \text{or center-of-mass \underline{even} if it acceleration.} \end{cases}$

Free-body diagram



$$\left.\begin{aligned} s &= R\theta \\ v &= R\omega \\ a &= R\alpha \end{aligned}\right] \text{ no slip}$$

$$I_{cm} = \frac{1}{2} m R^2$$

Steiner parallel Axis theorem

$$I_p = I_{cm} + m D^2$$

$$\sum \vec{\tau}_p = I_p \vec{\alpha}$$

$$f_s(0) + N(0) + mgR\sin\theta = \left[\frac{1}{2} mR^2 + mR^2\right] \frac{\vec{a}_{cm}}{R}$$

$$a_{cm} = \frac{2}{3} g \sin\theta$$