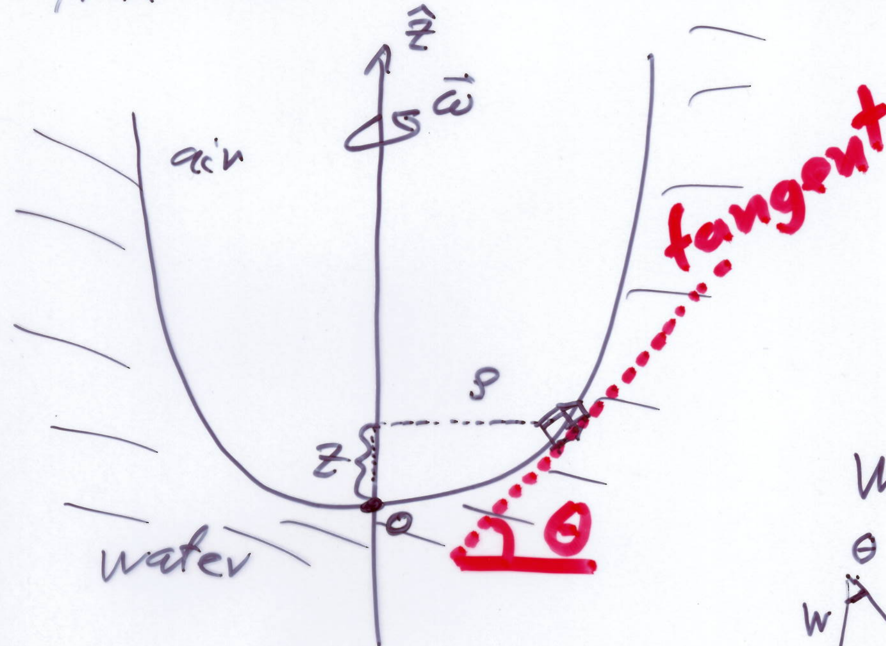
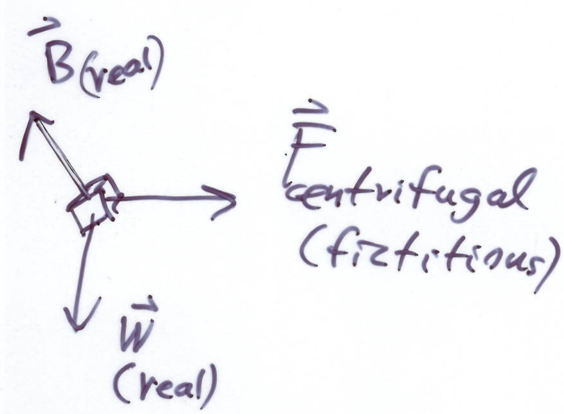
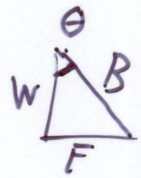
E.g. Shape of the water-air interface in a spinning container.

Non-inertial frame analysis





Weight



$\vec{a}_{Rot} = 0$

$$\tan\theta = \frac{opp}{adj} = \frac{F_{centif.}}{W} = \frac{\cancel{m}\,\omega^2 \rho}{\cancel{m}\,g} = \frac{dz}{d\rho}$$

$$dz = \frac{\omega^2 \rho\, d\rho}{g} \Rightarrow z = \frac{\omega^2 \rho^2}{2g} + \text{const}$$

parabola

Effective Potential Energy

$$V'_{eff} = mgz - \frac{m}{2}\omega^2 \rho^2$$

check: $-\vec{\nabla} V' = -mg\,\hat{z} + m\omega^2 \rho\,\hat{\rho} \equiv m\,\vec{g}_{eff}$

weight   centrifugal force

$\vec{g}_{eff} = \vec{g} - \vec{\omega} \times (\vec{\omega} \times \vec{r})$ coordinate-free

Water surface is a surface of constant V'_{eff}

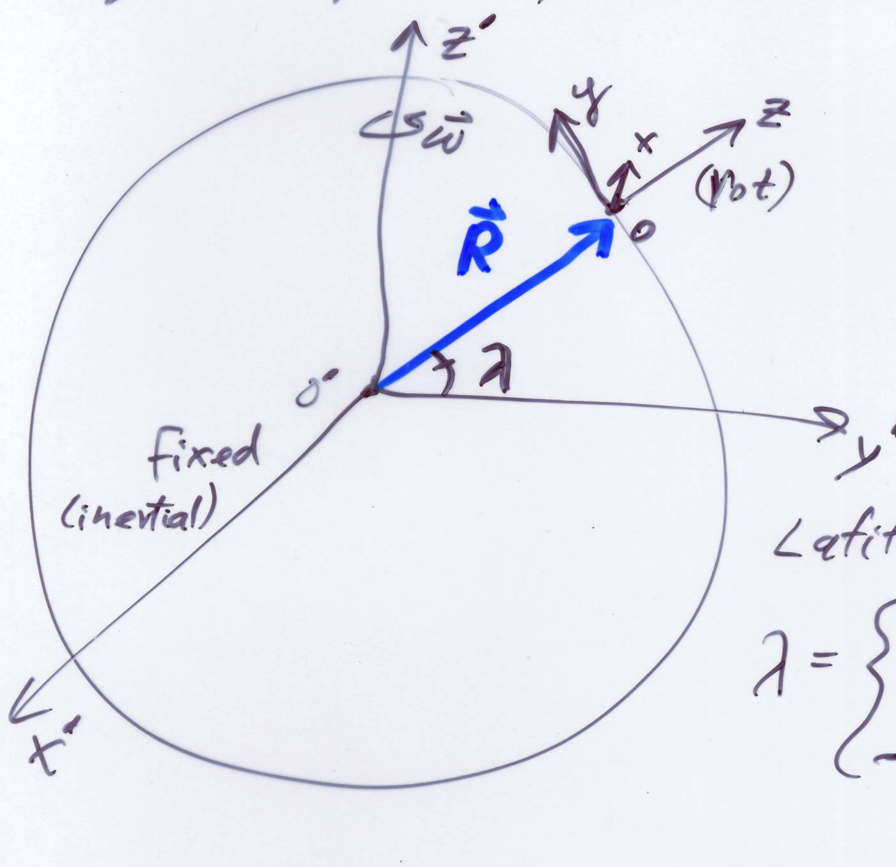
$\Rightarrow mgz - \frac{m}{2}\omega^2 \rho^2 = \text{constant}$ (parabola)

Spinning Earth

ellipsoid is called "geoid"
surface of constant V'_{eff}

<u>Motion close to the Earth's surface</u>

So acceleration due to gravity g does not change appreciably with altitude.



x - locally east
y - locally north
z - locally up

Latitude

$$\lambda = \begin{cases} +\frac{\pi}{2} & \text{North pole} \\ 0 & \text{equator} \\ -\frac{\pi}{2} & \text{South pole} \end{cases}$$

$$\vec{g}_{eff} = \vec{g} - \vec{\omega} \times [\vec{\omega} \times (\vec{r} + \vec{R})]$$

$$\sum \vec{F}_{eff} = \underset{\text{other than gravity}}{\sum \vec{F}_{real}} + m\vec{g} - \underline{m\ddot{\vec{R}}_{fixed}} - m\dot{\vec{\omega}} \times \vec{r}$$

ignore Earth's spin down

$$- m\vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2m\vec{\omega} \times \vec{v}_{rot}$$

Remember $\left(\frac{d\vec{Q}}{dt}\right)_{fixed} = \left(\frac{d\vec{Q}}{dt}\right)_{rot} + \vec{\omega} \times \vec{Q}$

$$\ddot{\vec{R}}_{fixed} = \left(\frac{d\dot{\vec{R}}}{dt}\right)_{fixed} = \cancel{\left(\frac{d\dot{\vec{R}}_{fix}}{dt}\right)_{rot}}^{\,0} + \vec{\omega} \times \dot{\vec{R}}_{fixed}$$

$\vec{R}$ does not change according to a rotating observer

$$= \vec{\omega} \times \left(\frac{d\vec{R}}{dt}\right)_{fixed} = \vec{\omega} \times \left[\cancel{\left(\frac{d\vec{R}}{dt}\right)_{rot}}^{\,0} + \vec{\omega} \times \vec{R}\right]$$

$$= \vec{\omega} \times (\vec{\omega} \times \vec{R})$$

$$\sum \vec{F}_{eff} = \underset{\text{not gravity}}{\sum \vec{F}_{real}} + m\vec{g} - m\vec{\omega} \times (\vec{\omega} \times \vec{R}) - m\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$- 2m\vec{\omega} \times \vec{v}_{rot}$$

$$\sum \vec{F}_{eff} = \sum_{\substack{\text{not}\\\text{gravity}}} \vec{F}_{real} + \underbrace{m\vec{g} - m\vec{\omega}\times\left[\vec{\omega}\times(\vec{r}+\vec{R})\right]}_{m\vec{g}_{eff}} - 2m\vec{\omega}\times\vec{v}_{rot}$$

$$\sum \vec{F}_{eff} = \sum_{\substack{\text{not}\\\text{gravity}}} \vec{F}_{real} + m\vec{g}_{eff} - 2m\vec{\omega}\times\vec{v}_{rot}$$

what a laboratory scale measures

E.g. Deflection of a particle dropped from rest from height h.

Rotating frame coordinates (unprimed)

$\vec{\omega} = \omega \hat{z}'$ ← in inertial frame

$$\left.\begin{aligned} \omega_x &= 0 \\ \omega_y &= \omega \cos\lambda \\ \omega_z &= \omega \sin\lambda \end{aligned}\right\} \text{in rot frame} \qquad \left\{\begin{aligned} v_x^{rot} &= \mathcal{O}(\omega) \\ v_y^{rot} &= \mathcal{O}(\omega) \\ v_z^{rot} &= -g_{eff}\,t \approx -gt \end{aligned}\right.$$

first order in ω

zeroth order in ω

Coriolis acceleration $-2\vec{\omega}\times\vec{v}_{rot} = \vec{a}^{rot}$

$$= -2\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \omega\cos\lambda & \omega\sin\lambda \\ 0 & 0 & -gt \end{vmatrix} = 2\omega g t\cos\lambda\,\hat{x}$$

$$a_x^{rot} = 2\omega g t\cos\lambda + O(\omega^2) = \ddot{x}$$

integrate twice to get deflection Δx

$$a_y^{rot} = O(\omega^2)$$

$$a_z^{rot} = -g + O(\omega^2)$$

get free-fall time t

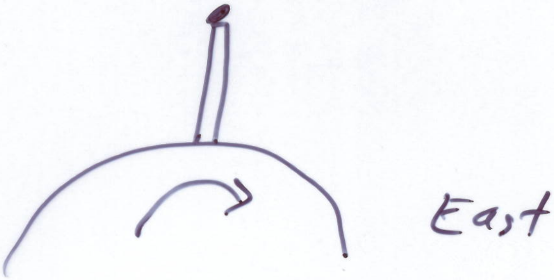
Free-fall time $h = \frac{1}{2}gt^2 + O(\omega)$

$$t = \sqrt{\frac{2h}{g}} + O(\omega)$$

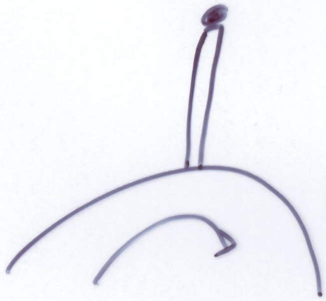
$$v_x^{rot} = \int_{t'=0}^{t} a_x^{rot}(t')\,dt' = \int_{t'=0}^{t} 2\omega g t\cos\lambda = \omega g\cos\lambda\, t^2 + \cancel{v_{0}}$$

$$\Delta x = \int_{t''=0}^{t} v_x^{rot}(t'')\,dt'' = \frac{1}{3}\omega g\cos\lambda\, t^3 = \frac{1}{3}\omega g\cos\lambda\left(\frac{2h}{g}\right)^{3/2}$$

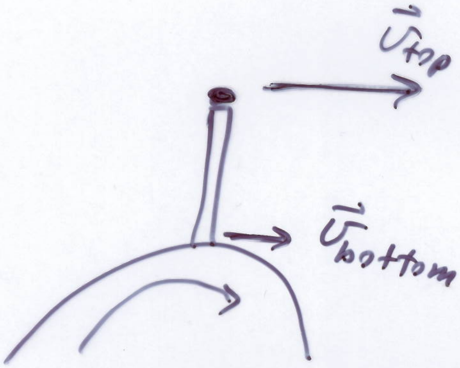
East



Aristotle



Galileo



Coriolis