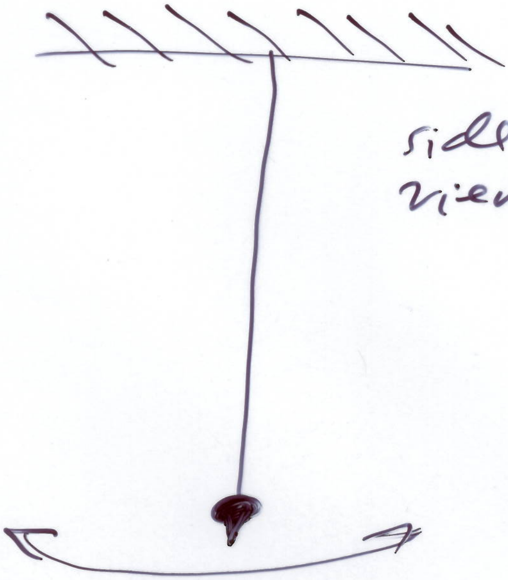
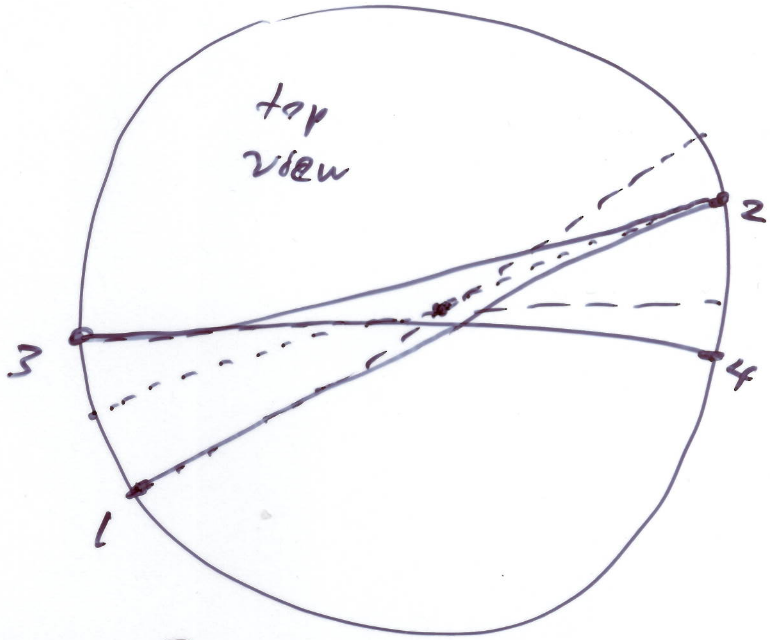
Foucault's Pendulum

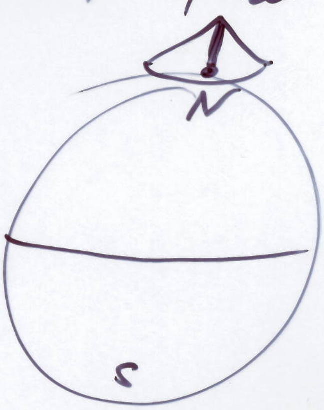




Northern Hemisphere - Coriolis deflection to the right

Plane of pendulum rotates (precesses) clockwise as seen from above.

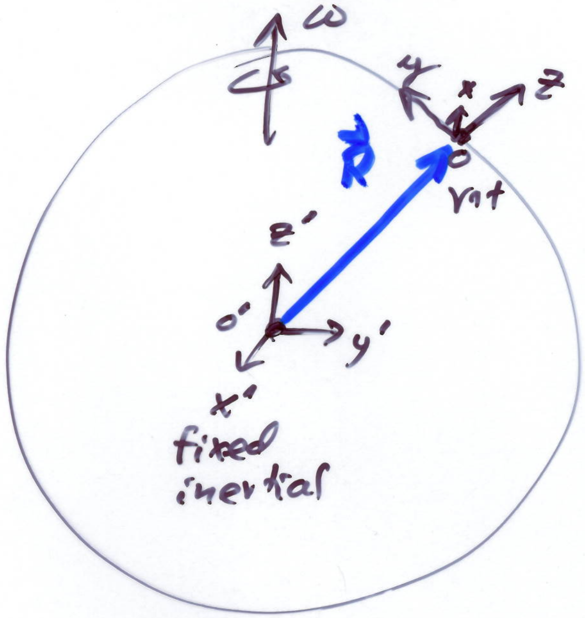
Proof that the Earth rotates.

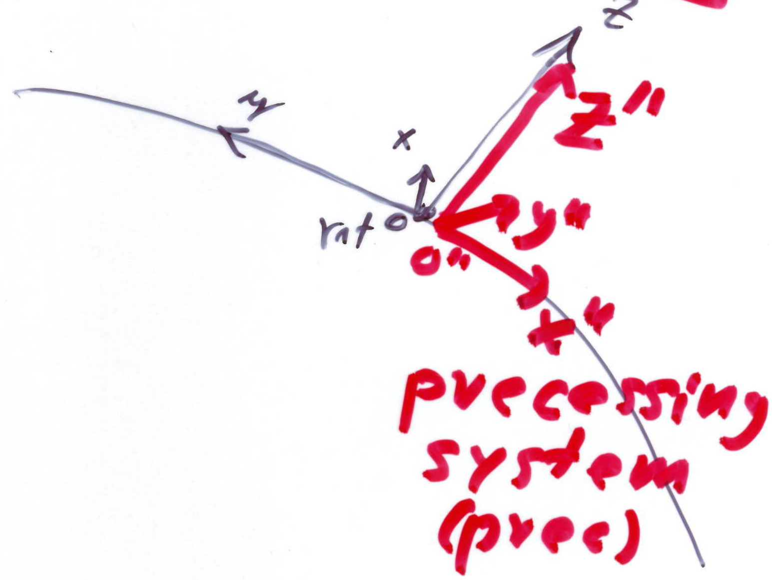


At the North Pole, a Foucault Pendulum will precess once per day.

On the equator, the pendulum will not precess at all.

5b.





$$\sum_{rot} \vec{F}_{eff} = \underset{(real)}{\vec{T}} + m\vec{g}_{eff} - 2m\vec{\omega} \times \vec{v}_{rot}$$

The (rot) and (prec) have the same origin.
Call the vector from O to O'', $\vec{P}$

$$\vec{P} = 0 , \quad \dot{\vec{P}} = 0 , \quad \ddot{\vec{P}} = 0$$

The angular velocity of the precessing frame is $\vec{\Omega}$ (locally down ($-\hat{z}$) in Northern Hemi.)

$$\vec{\Omega} = \text{constant} \Rightarrow \dot{\vec{\Omega}} = 0$$

Remeber: $\left(\frac{d\vec{r}}{dt}\right)_{fixed} = \left(\frac{d\vec{r}}{dt}\right)_{rot} + \vec{\omega} \times \vec{r}$

Also for any vector $\vec{Q}$

By analogy

$$\left(\frac{d\vec{r}}{dt}\right)_{rot} = \left(\frac{d\vec{r}}{dt}\right)_{prec} + \vec{\Omega} \times \vec{r}$$

$$\vec{v}_{rot} = \vec{v}_{prec} + \vec{\Omega} \times \vec{r}$$

$$\left(\frac{d\vec{v}_{rot}}{dt}\right)_{rot} = \vec{a}_{rot} = \frac{d}{dt}\left[\vec{v}_{prec} + \vec{\Omega} \times \vec{r}\right]_{rot}$$

$$= \frac{d}{dt}\left[\vec{v}_{prec} + \vec{\Omega} \times \vec{r}\right]_{prec} + \vec{\Omega} \times \left[\vec{v}_{prec} + \vec{\Omega} \times \vec{r}\right]$$

$$= \left(\frac{d\vec{v}_{prec}}{dt}\right)_{prec} + \cancelto{0}{\dot{\vec{\Omega}} \times \vec{r}} + 2\vec{\Omega} \times \vec{v}_{prec} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$$

$$= \vec{a}_{prec} + \underbrace{2\vec{\Omega} \times \vec{v}_{prec}}_{\text{Coriolis}} + \underbrace{\vec{\Omega} + (\vec{\Omega} \times \vec{r})}_{\text{centrifugal}}$$

$$\sum_{prec} \vec{F}_{eff} = \sum_{rot} \vec{F}_{eff} - 2m\vec{\Omega}\times\vec{v}_{prec} - m\vec{\Omega}\times(\vec{\Omega}\times\vec{r})$$

$$\hookrightarrow \vec{v}_{rot} - \vec{\Omega}\times\vec{r}$$

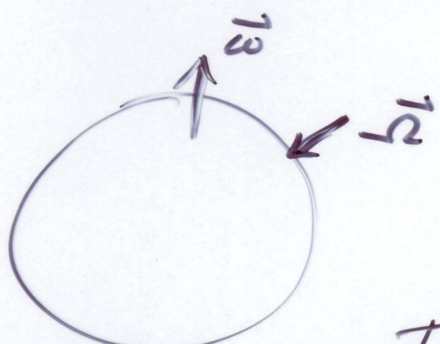
$$\hookrightarrow \vec{T} + m\vec{g}_{eff} - 2m\vec{\omega}\times\vec{v}_{rot}$$

$$\sum_{prec} \vec{F}_{eff} = \vec{T} + m\vec{g}_{eff} - 2m(\vec{\omega}+\vec{\Omega})\times\vec{v}_{rot} + m\vec{\Omega}\times(\vec{\Omega}\times\vec{r})$$

An observer in the precessing frame can move with the plane of the Foucault pendulum.

$\Rightarrow$ will see no ~~pre~~precession $\Rightarrow$ no Coriolis force

$$\Rightarrow -2m(\vec{\omega}+\vec{\Omega})\times\vec{v}_{rot} = 0$$



$\vec{v}_{rot}$ has only x and y components (locally East-West or North-South)

Then $(\vec{\omega}+\vec{\Omega})$ can have zero cross product with $\vec{v}_{rot}$ if

$$|\vec{\Omega}| = |\vec{\omega}|\sin\lambda$$

Check: N. pole $\lambda = +\frac{\pi}{2}$ $\Rightarrow \Omega = \omega$

Equator: $\lambda = 0$ $\Rightarrow \Omega = 0$

Vectors, Tensors, Scalars, & Rotation

<u>def</u> scalar – a quantity that does not change when the coordinate system is rotated.

e.g. Temp., Pressure, mass, time...

<u>def</u> vector – a quantity that changes like displacement $\vec{r}$ under a rotation.

<u>Notation</u> $\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ 3×1 matrix

Transpose $\vec{v}^{\,T} = (v_1, v_2, v_3) \Rightarrow$ 1×3 matrix

Components v_i ← index

vectors have 1 index
scalar have no indices

Vectors have "rank" = 1
Scalars have "rank" = 0

$i = \{1, \dots d\}$ ↑ dimension of space

Scalars ~~be~~ are rank-0 tensors
vectors are rank-1 tensors
rank-n tensors