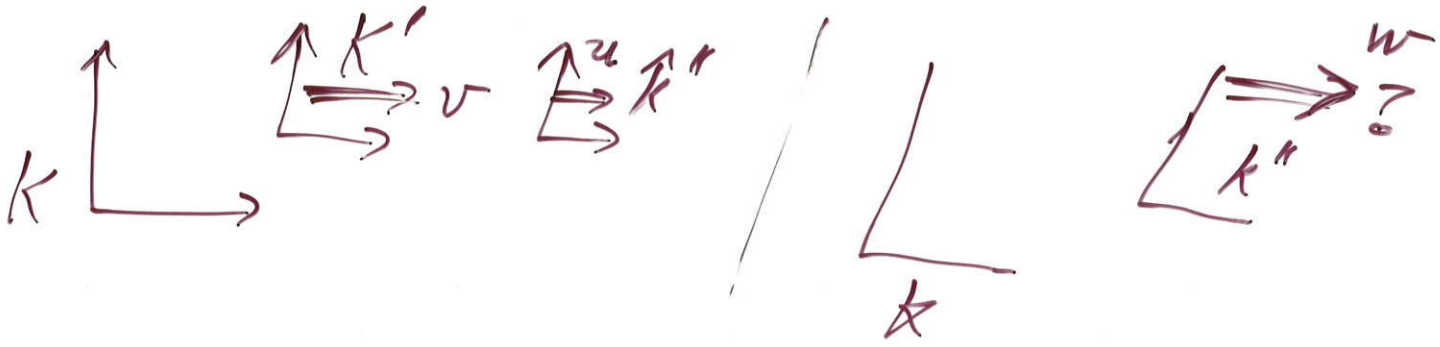

$$W = v + u$$ Galilean / Newtonian

$$\frac{W}{c} = \frac{\frac{v}{c} + \frac{u}{c}}{1 + \frac{u \cdot v}{c^2}}$$ Einstein

Notice

$$\tanh(\alpha + \beta) = \frac{\tanh(\alpha) + \tanh(\beta)}{1 + \tanh(\alpha)\tanh(\beta)}$$

$\alpha + \beta = \gamma$

$$\left.\begin{aligned} \alpha &\equiv \operatorname{arctanh}\left(\tfrac{v}{c}\right) \\ \beta &\equiv \operatorname{arctanh}\left(\tfrac{u}{c}\right) \\ \gamma &= \operatorname{arctanh}\left(\tfrac{w}{c}\right) \end{aligned}\right\}$$ rapidity for relativists

---

Rapidity for particle Physicists

$$\phi = \operatorname{arctanh}\left(\frac{|p|c}{E}\right) = \frac{1}{2}\ln\left(\frac{E + |p|c}{E - |p|c}\right)$$

$$\phi' = \frac{1}{2} \ln\left(\frac{E + P_{along}c}{E - P_{along}c}\right)$$

pseudo rapidity

$$\eta = \frac{1}{2} \ln\left(\frac{|P| + P_{along}}{|P| - P_{along}}\right)$$

$$|\vec{P}| = \sqrt{P_x^2 + P_y^2 + P_z^2}$$

$\vec{P}$ is the space part of the 4- momentum p^μ

$$g_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$cp^\mu = (E, cP_x, cP_y, cP_z)$$

$$cp_\mu = (E, -cP_x, -cP_y, -cP_z)$$

$$\partial_\mu = (c\partial_t, \partial_x, \partial_y, \partial_z) = \left(c\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

SO(3) rotation by angle θ about the z-axis. θ_{CCW} as you look down the z-axis

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$x' = \cos\theta\, x - \sin\theta\, y < x$$

Rapidity α boost along x-axis

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cosh(\alpha) & -\sinh(\alpha) & 0 & 0 \\ -\sinh(\alpha) & \cosh(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$E = mc^2 \cosh(\alpha)$$

$$|\vec{p}| = mc \sinh(\alpha)$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\cosh^2\alpha - \sinh^2(\alpha) = 1$$

$$\frac{E^2}{(mc^2)^2} - \frac{|\vec{p}|^2}{(mc)^2} = 1$$

$$E^2 = |\vec{p}|^2 c^2 + m^2 c^4$$