Class 2, The Schrodinger Eqn and Expectation Values

Thursday, August 23, 2018 8:01 AM

$$\ast \quad \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{x})\right)\underbrace{\Psi(\vec{x},t)} = i\hbar\frac{\partial}{\partial t}\Psi(\vec{x},t) \quad \ast$$

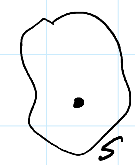
$|\Psi(\vec{x},t)|^2 \Delta V$ = Prob. a pcle is found in the volume ΔV

$$\int |\Psi(\vec{x},t)|^2 d^3x = 1 \qquad \text{"Normalization Condition"}$$

$$|\Psi(\vec{x},t)|^2 = \rho(\vec{x},t)$$

$$\int \rho(\vec{x},t)\, d^3x = 1$$

$$\frac{\partial\rho}{\partial t}(\vec{x},t) \neq 0$$



$$\frac{\partial}{\partial t}\int \Psi^*(\vec{x},t)\,\Psi(x,t)\, d^3x = 0$$

$$\int\left(\underbrace{\frac{\partial}{\partial t}\Psi^*(\vec{x},t)}\,\Psi(x,t) + \Psi^*(\vec{x},t)\underbrace{\frac{\partial}{\partial t}\Psi(\vec{x},t)}\right)d^3x$$

$$\frac{\partial\Psi(\vec{x},t)}{\partial t} = -\frac{i}{\hbar}\left[-\frac{\hbar^2}{2m}\nabla^2\Psi(x,t) + V(\vec{x})\,\Psi(\vec{x},t)\right]$$

$$\frac{\partial\Psi^*}{\partial t} = +\frac{i}{\hbar}\left[-\frac{\hbar^2}{2m}\nabla^2\Psi^*(\vec{x},t) + V^*(\vec{x})\,\Psi^*(\vec{x},t)\right]$$

$$\Rightarrow \int\left\{-\frac{\hbar^2}{2m}\left[\left(\nabla^2\Psi^*(\vec{x},t)\right)\Psi(\vec{x},t) - \Psi^*(\vec{x},t)\,\nabla^2\Psi(\vec{x},t)\right] + \left(V^*(\vec{x}) - V(\vec{x})\right)\Psi^*(\vec{x},t)\Psi(\vec{x},t)\right\}d^3x = 0$$

$\ast$ Take $V(\vec{x})$ to be real, $\ast$

$$\Rightarrow$$

$$-\frac{\hbar^2}{2m}\int\left[\left(\nabla^2\Psi^*\right)\Psi - \Psi^*\vec{\nabla}^2\Psi\right]d^3x = 0$$

$$\Psi^*\nabla^2\Psi = \vec{\nabla}\cdot\left(\Psi^*\vec{\nabla}\Psi\right) - \left(\vec{\nabla}\Psi^*\right)\cdot\left(\vec{\nabla}\Psi\right)$$

$$\left(\nabla^2\Psi^*\right)\Psi = \vec{\nabla}\cdot\left(\left(\vec{\nabla}\Psi^*\right)\Psi\right) - \left(\vec{\nabla}\Psi^*\right)\cdot\left(\vec{\nabla}\Psi\right)$$

$$-\frac{\hbar^2}{2m}\int\vec{\nabla}\cdot\left[\Psi^*\vec{\nabla}\Psi - \left(\vec{\nabla}\Psi^*\right)\Psi\right]d^3x = 0$$

$$-\frac{\hbar^2}{2m}\int \vec{\nabla}\cdot\left[\Psi^*\vec{\nabla}\Psi - (\vec{\nabla}\Psi^*)\Psi\right] d^3x = 0$$

$$\int \vec{\nabla}\cdot\vec{J}\, d^3x = 0 \qquad \vec{J} = \frac{i\hbar}{2m}\left[\Psi^*(\vec{x},t)\vec{\nabla}\Psi - (\vec{\nabla}\Psi^*(\vec{x},t))\Psi(\vec{x},t)\right]$$

$$\int\left(\frac{\partial}{\partial t}\rho\right) d^3x = 0$$

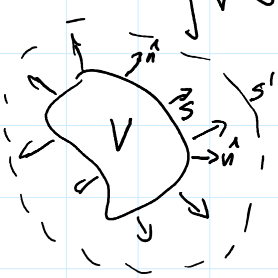
$$\int\left(\frac{\partial}{\partial t}\rho + \vec{\nabla}\cdot\vec{J}\right) d^3x = 0 \qquad \frac{\partial}{\partial t}\rho(\vec{x},t) + \vec{\nabla}\cdot\vec{J} = 0$$

$$\rho(\vec{x},t) = \Psi^*\Psi \qquad \frac{\partial\rho}{\partial t} = -\vec{\nabla}\cdot\vec{J}$$

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\int\vec{\nabla}\cdot\vec{J}\, d^3x = \int\underbrace{(\vec{J}\cdot\hat{n})}\, dS$$

$$= 0$$



$\Longrightarrow$ The Schroedinger Eqn conserves particle #

Define Expectation Value of the position:

$$\langle\vec{x}\rangle \equiv \int \vec{x}\,|\Psi(\vec{x},t)|^2\, d^3x$$

In one-d

$$\langle x\rangle = \int x\,|\Psi(x,t)|^2\, dx$$

$\hookrightarrow$ "Ensemble Average"

$$\frac{d\langle\hat{x}\rangle}{dt} \Rightarrow \hat{P}_x = -i\hbar\frac{\partial}{\partial x}$$

$$\frac{d\langle \hat{x} \rangle}{dt} = \frac{1}{m} \langle \hat{p}_x \rangle$$