Class 19 Coherent State Continued

Tuesday, November 6, 2018 9:02 AM

We have found, that

$$\psi_\alpha(x) = D_\alpha \psi_0(x)$$

$$\hat{a}\,\psi_\alpha(x) = \alpha\,\psi_\alpha(x)$$

$$D_\alpha = e^{(\alpha a^+ - \alpha^* a)}$$

$$D_\alpha \psi_0(x) = e^{(\alpha a^+ - \alpha^* a)} \psi_0(x)$$

Note, $e^{(\hat{A}+\hat{B})} \neq e^{\hat{A}} e^{\hat{B}}$

For Homework you will show that,

$$e^{(\hat{A}+\hat{B})} = e^{\hat{A}} e^{\hat{B}} e^{-1/2[\hat{A},\hat{B}]} + \underbrace{\cdots\cdots}_{\text{are zero if } [\hat{A},\hat{B}] = \#}$$

for $\hat{A} = \alpha a^+$ & $\hat{B} = \alpha^* a$

$$[\hat{A}, \hat{B}] = |\alpha|^2 [a^+, a] = -|\alpha|^2$$

$$e^{(\alpha a^+ - \alpha^* a)} = e^{\alpha \hat{a}^+} e^{\alpha^* \hat{a}} e^{-|\alpha|^2/2}$$

$$e^{(\alpha a^+ - \alpha^* a)} \psi_0(x) = e^{-|\alpha|^2/2} e^{\alpha \hat{a}^+} \psi_0(x)$$

$$\psi_\alpha(x) = e^{-|\alpha|^2/2} e^{\alpha \hat{a}^+} \psi_0(x) \quad ; \qquad \psi_0(x) = \left(\frac{m\omega}{2\hbar}\right)^{1/4} e^{-\sqrt{\frac{m\omega}{\hbar}}\, x^2/2}$$

$$\psi_\alpha(x) = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{n!} \underbrace{(\hat{a}^+)^n \psi_0(x)}_{\sqrt{n!}\,\psi_n(x)} \quad \#$$

$$\psi_\alpha(x) = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} \psi_n(x) = \sum_n c_n \psi_n(x) \quad ; \quad c_n = \frac{e^{-|\alpha|^2/2}\alpha^n}{\sqrt{n!}}$$

$$\langle n \rangle = \sum_n n |c_n|^2 = e^{-|\alpha|^2} \sum_{n=0}^{\infty} n \frac{|\alpha|^{2n}}{n!}$$

$$= e^{-|\alpha|^2} \sum \frac{1}{(n-1)!} \alpha^{2n} \qquad m = n-1, \quad n = m+1$$

$$= e^{-|\alpha|^2} \sum_{n=1} \frac{1}{(n-1)!} \alpha^{2n}$$

$m = n-1$

$n = m+1$

$$= e^{-|\alpha|^2} \sum_{m} \frac{1}{m!} |\alpha|^{2(m+1)}$$

$$= |\alpha|^2 e^{-|\alpha|^2} \underbrace{\sum_{m} \frac{\left(|\alpha|^2\right)^m}{m!}}_{e^{|\alpha|^2}}$$

$$\therefore \langle n \rangle = |\alpha|^2$$

$$\Delta n = \left[\langle n^2 \rangle - \langle n \rangle^2 \right]^{1/2} = \langle n \rangle = |\alpha|^2$$

$$|c_n|^2 = \frac{e^{-|\alpha|^2} |\alpha|^{2n}}{n!} \quad : \quad \text{"Poisson Distribution"}$$

$$\frac{e^{-\lambda} \lambda^n}{n!} \; ; \quad \lambda = |\alpha|^2$$

As n gets large the Poisson distribution turns into the gaussian Distribution.