Class 20 Dirac Formalism

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Dirac Formalism:

For the H.O. the eigenfunctions of $\hat{H}$ are eigenfunctions of the Number operator $\hat{N} = \hat{a}^{+}\hat{a}$

$$\underset{\sim}{a} = \begin{array}{c|cccc} m \backslash n & 0 & 1 & 2 & \cdots \\ \hline 0 & \multicolumn{4}{l}{\begin{pmatrix} 0 & 0 & & \\ & 0 & \sqrt{1} & \\ & & 0 & \sqrt{2} \\ & & & \ddots \end{pmatrix}} \\ 1 & & & & \\ 2 & & & & \\ \vdots & & & & \end{array} \quad \begin{pmatrix} \\ \\ \end{pmatrix}$$

$$\underset{\sim}{a}_{mn} = \int \psi_m^*(x)\, \hat{a}\, \psi_n(x)\, dx$$

$\uparrow$ Rows

$$= \sqrt{n-1} \int \psi_m(x)\, \psi_{n-1}(x)\, dx$$

$$= \sqrt{n-1}\ \delta_{m\, n-1}$$

$$a\psi \sim \underset{\sim}{a}\,\underset{\sim}{V}$$

To each quantum state the corresponds state vector.

$$\hat{Q}\,|\psi\rangle = |\psi'\rangle \qquad |\psi\rangle \sim \vec{V}$$

$\longrightarrow$ "Ket Vectors"

$$\hat{Q}\,|\psi\rangle = q\,|\psi\rangle \longrightarrow \hat{Q}\,|q\rangle = q\,|q\rangle$$

$\uparrow$ Hermitian $\quad \uparrow$ Real

$$\underbrace{\hat{H}\,|E\rangle = E\,|E\rangle}$$

$$\hat{H}\,\psi_E(x) = E\,\psi_E(x) \qquad \langle q'|q\rangle = 0 \quad q' \neq q$$

$$\langle \varphi|\psi\rangle \equiv \int \varphi^*(x)\,\psi(x)\,dx$$

$$\hat{X}|x\rangle = x|x\rangle \quad , \quad \hat{P}|p\rangle = p|p\rangle$$

$$\text{Quantum State} \longleftrightarrow |\psi\rangle \, , \, \langle\psi|$$

$$\langle \varphi|\psi\rangle = \int \varphi^*(x)\,\psi(x)\,dx$$

$$\psi(x) = \sum c_n \psi_n(x) \quad , \quad \varphi(x) = \sum d_n \psi_n(x)$$

$$= \int \sum_{m,n} d_m^* \psi_m^*(x)\, c_n \psi_n(x)\, dx$$

$$= \sum_{m,n} d_m^* c_n \underbrace{\int \psi_m^*(x)\,\psi_n(x)\,dx}_{\delta_{mn}}$$

$$= \sum_m d_m^* c_m$$

$$= \left(\underset{\sim}{d}^{T*}\right)\left(\underset{\sim}{c}\right)$$

$$\langle \varphi | \psi \rangle = \underset{\sim}{d}^* \cdot \underset{\sim}{c}$$

$$\underset{\sim}{d} = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \end{pmatrix} \qquad \underset{\sim}{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \end{pmatrix}$$

$$\downarrow$$

$$|\psi\rangle \sim \begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix}$$

$$|\varphi\rangle = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \end{pmatrix}$$