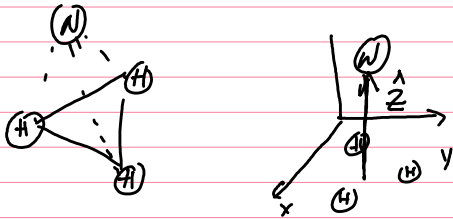


Two State System

Example of a Two State System

Ammonia Molecule:



A general two state system is characterized by two basis vectors: $|1\rangle, |2\rangle$

We assume $\langle 1|2\rangle = 0$; $\langle 1|1\rangle = 1$; $\langle 2|2\rangle = 1$

The most general quantum state:

$$|\psi\rangle = c_1|1\rangle + c_2|2\rangle = \sum_n c_n|n\rangle$$

We can define the exchange operator:

$$\hat{E} \ni \hat{E}|1\rangle = |2\rangle \quad \hat{E}|2\rangle = |1\rangle$$

$$\hat{E}^2 = \mathbb{1} = |1\rangle\langle 1| + |2\rangle\langle 2|$$

$$\hat{E} = \sum_{i,j} E_{ij} |i\rangle\langle j| = \begin{matrix} \overset{1}{\downarrow} & \overset{2}{\downarrow} \\ E_{11} & E_{12} \\ E_{21} & E_{22} \end{matrix} |1\rangle\langle 1| + |2\rangle\langle 2|$$

$$\hat{E} = |1\rangle\langle 2| + |2\rangle\langle 1|$$

$$\hat{E}^2 = \mathbb{1} \quad (\text{check})$$