

vzp = {1, 0, 0}; vz0 = {0, 1, 0}; vzm = {0, 0, 1};

Lz = h {{1, 0, 0}, {0, 0, 0}, {0, 0, -1}}; Lz // MatrixForm

$$\begin{pmatrix} h & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -h \end{pmatrix}$$

Lx = h / Sqrt[2] {{0, 1, 0}, {1, 0, 1}, {0, 1, 0}}; Lx // MatrixForm

$$\begin{pmatrix} 0 & \frac{h}{\sqrt{2}} & 0 \\ \frac{h}{\sqrt{2}} & 0 & \frac{h}{\sqrt{2}} \\ 0 & \frac{h}{\sqrt{2}} & 0 \end{pmatrix}$$

Ly = h / Sqrt[2] / I {{0, 1, 0}, {-1, 0, 1}, {0, -1, 0}};

Ly // MatrixForm

$$\begin{pmatrix} 0 & -\frac{i h}{\sqrt{2}} & 0 \\ \frac{i h}{\sqrt{2}} & 0 & -\frac{i h}{\sqrt{2}} \\ 0 & \frac{i h}{\sqrt{2}} & 0 \end{pmatrix}$$

L2 = Lx.Lx + Ly.Ly + Lz.Lz; L2 // MatrixForm

$$\begin{pmatrix} 2 h^2 & 0 & 0 \\ 0 & 2 h^2 & 0 \\ 0 & 0 & 2 h^2 \end{pmatrix}$$

Lz.vzp

{h, 0, 0}

Lz.vz0

{0, 0, 0}

Lz.vzm

{0, 0, -h}

Eigensystem[Lx]

{{{0, -h, h}, {-1, 0, 1}, {1, -\sqrt{2}, 1}, {1, \sqrt{2}, 1}}}

$$\mathbf{vxp} = \{1, \sqrt{2}, 1\} / \text{sqrt}[\{1, \sqrt{2}, 1\} \cdot \{1, \sqrt{2}, 1\}]$$

$$\left\{ \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2} \right\}$$

$$\mathbf{vxm} = \{1, -\sqrt{2}, 1\} / \text{sqrt}[\{1, -\sqrt{2}, 1\} \cdot \{1, -\sqrt{2}, 1\}]$$

$$\left\{ \frac{1}{2}, -\frac{1}{\sqrt{2}}, \frac{1}{2} \right\}$$

$$\mathbf{vx0} = \{-1, 0, 1\} / \text{sqrt}[\{-1, 0, 1\} \cdot \{-1, 0, 1\}]$$

$$\left\{ -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\}$$

$$\mathbf{vxp.vz0}$$

$$\frac{1}{\sqrt{2}}$$

$$\mathbf{vx0.vz0}$$

$$0$$

$$\mathbf{vxm.vz0}$$

$$-\frac{1}{\sqrt{2}}$$

$$\mathbf{vxp.vzp}$$

$$\frac{1}{2}$$

$$\mathbf{vx0.vzp}$$

$$-\frac{1}{\sqrt{2}}$$

$$\mathbf{vxm.vzp}$$

$$\frac{1}{2}$$

$$\mathbf{vxp.vzm}$$

$$\frac{1}{2}$$

$$\mathbf{vx0.vzm}$$

$$\frac{1}{\sqrt{2}}$$

vxm.vzm

$$\frac{1}{2}$$

Lx.vxp == h vxp

True

Lx.vxm == -h vxm

True

Lx.vx0 == 0 vx0

True

Lx.Ly - Ly.Lx == I h Lz

True