When a measurement of an observable $\hat{A}$ is made on a generic state $\left| \psi \right>$, the probability of obtaining eigenvalue $\lambda_n$ is $|\left< x_n | \psi \right>|^2$ where $\hat{A}/x_n = \lambda_n \hat{A}$.

- Normalized $\left< \psi | \psi \right> = 1$, $\left< x_i | x_n \right> = \delta_{in}$

- $\left< x_n | \psi \right>$ is a complex number called the "probability amplitude" or "square root of probability."

- $\left| \psi \right> = \sum_n c_n \left| x_n \right> \quad$ (left multiply by $\left< x_i |$)

$$\left< x_j | \psi \right> = \sum_n c_n \left< x_j | x_n \right> = \sum_n c_n \delta_{jn} = c_j$$

- Change $c_n = \left< x_n | \psi \right>$ substitute

$$\left| \psi \right> = \sum_n \left( \left< x_n | \psi \right> \right) \left| x_n \right> = \sum_n \left( x_n \left< x_n | \psi \right> \right) \left| x_n \right>$$

- Completeness: $\sum_n \left| x_n \right> \left< x_n \right> = \hat{1}$
The probability of getting some result is unity

\[ 1 = |<\psi|\chi>|^2 = \sum_m \sum_n C_m^* C_n \langle X_m | X_n \rangle \]

\[ = \sum_n |C_n|^2 = 1 \]

\[ |C_n|^2 = C_n^* C_n \quad \text{Real} \]

Expectation value of \( \hat{A} \) in the state \( |\psi> \)

\[ \langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle = (\sum_m C_m^* X_m) \hat{A} (\sum_n C_n X_n) \]

\[ = \sum_m \sum_n C_m^* C_n a_n \langle X_m | X_n \rangle = \sum_n |C_n|^2 a_n \]

5 Immediately after measurement of \( \hat{A} \) has yielded an eigenvalue \( a_n \), the state of the system is the normalized eigenstate \( |\chi_n> \).

"collapse of the wavefunction" violates unitarity.

not deterministic, but probabilistic.

Solution is called decoherence.
The time evolution of a quantum system preserves the normalization of the ket $|\Psi(t)\rangle$.

$$|\Psi(t)\rangle = \hat{U}(t,t_0) |\Psi(t_0)\rangle$$

where $\hat{U}$ is a unitary operator. $\hat{U}^+ \hat{U} = \mathbb{1}$

$\hat{U}^+$ hermitian conjugate, dagger, adjoint

This postulate implies the Schrödinger Equation,

$$i\hbar \frac{d}{dt} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle$$

where $\hat{H}$ is a hermitian op. ($\hat{H}^+ = \hat{H}$)

- $|\Psi(t)\rangle = \hat{U}(t,t_0) |\Psi(t_0)\rangle$ evolve deterministically forward and backward in time.

- Unitarity $\hat{U}^+(t,t_0) \hat{U}(t,t_0) = \mathbb{1}$

- Norm of state is preserved

$$\langle \Psi(t) | \Psi(t) \rangle = \langle \Psi(t_0) | \hat{U}^+(t,t_0) \hat{U}(t,t_0) | \Psi(t_0) \rangle$$

$$= \langle \Psi(t_0) | \Psi(t_0) \rangle$$
- Identity: \( \hat{U}(t_0, t_0) = \mathbb{1} \) \( \forall t_0 \)

- Composition

\[
|\Psi(t_2)\rangle = \hat{U}(t_2, t_1) |\Psi(t_1)\rangle = \hat{U}(t_2, t_0) \hat{U}(t_1, t_0) |\Psi(t_0)\rangle = \hat{U}(t_2, t_0) |\Psi(t_0)\rangle
\]

\[
\Rightarrow \hat{U}(t_2, t_0) = \hat{U}(t_2, t_1) \hat{U}(t_1, t_0)
\]

-Inverse: \( \hat{U}(t_0, t) \hat{U}(t, t_0) = \mathbb{1} \)

\[
\Rightarrow \hat{U}(t_0, t) = \left[ \hat{U}(t, t_0) \right]^{-1} = \hat{U}^+(t, t_0)
\]

\[
\frac{\partial}{\partial t} |\Psi(t)\rangle = \left[ \frac{\partial \hat{U}(t, t_0)}{\partial t} \right] |\Psi(t_0)\rangle = \left[ \frac{\partial \hat{U}^+(t, t_0)}{\partial t} \right] |\Psi(t)\rangle
\]

\[
\Rightarrow \hat{A}(t, t_0)
\]
$\hat{A}(t, t_0)$ is anti hermitian

$\hat{O}(t, t_0) \hat{O}^+(t, t_0) = \hat{I}$

time derivative both sides,

$\left[ \frac{\partial \hat{O}(t, t_0)}{\partial t} \right] \hat{O}^+(t, t_0) + \hat{O}(t, t_0) \frac{\partial \hat{O}^+(t, t_0)}{\partial t} = 0$

$\hat{A}(t, t_0) + \hat{A}^+(t, t_0) = 0$

$\Rightarrow \hat{A}(t, t_0) = -\hat{A}^+(t, t_0)$

$\Rightarrow \hat{A}(t, t_0)$ is independent of $t_0$.

$\hat{A}(t, t_0) = \left[ \frac{\partial \hat{O}(t, t_0)}{\partial t} \right] \hat{O}^+(t, t_0)$

insert $\hat{I} = \hat{O}(t_0, t) \hat{O}^+(t, t_0)$

$= \left[ \frac{\partial \hat{O}(t, t_0)}{\partial t} \right] \hat{O}^+(t, t_0)$

$= \hat{A}(t, t_0)$

$\hat{A}(t, t_1) = \hat{A}(t)$
\[ \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{A}(t) |\psi(t)\rangle \quad \hat{A} \text{ is anti-hermitian} \]

Define \[ \hat{H}(t) = \text{dim} \left[ \frac{\partial \Omega(t, t_0)}{\partial t} \right] U^*(t, t_0) \]
\[ \text{dimension, units of energy}. \]

\[ \text{dim} \left[ \frac{\partial \Omega(t, t_0)}{\partial t} \right] U^*(t, t_0) \]
\[ \text{dimension, units of energy}. \]

\[ \text{dim} \left[ \frac{\partial \Omega(t, t_0)}{\partial t} \right] \]
\[ \text{dimension, units of energy}. \]

\[ \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle \quad \text{Schrödinger equation} \]

Why is $\hat{H}$ the Hamiltonian (energy)?