Why is $\hat{H}$ the Hamiltonian?

① $\left[\begin{array}{l} \dfrac{\partial}{\partial x} \to \text{momentum} \\[2ex] \dfrac{\partial}{\partial t} \to \text{energy} \end{array}\right.$

$$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x} \qquad \hat{H} = i\hbar\frac{\partial}{\partial t}$$

$$g = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix}$$

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② $\hat{H}$ plays the same role as classical Hamiltonian from mechanics.

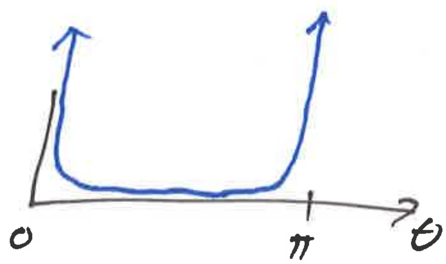
$$\frac{d}{dt}\langle\hat{B}\rangle \sim [\hat{H}, \hat{B}] \xrightarrow[\text{classical}]{} \{H, B\}$$

Poisson brackets

$\psi(x) = \langle x | \psi \rangle$ projection of ket $|\psi\rangle$ on $|x\rangle$ basis

must be single-valued (function)

$\psi(x)$ need not be finite

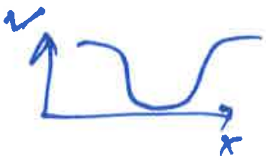


$dP = |\psi|^2 dV$ ← goes to zero.

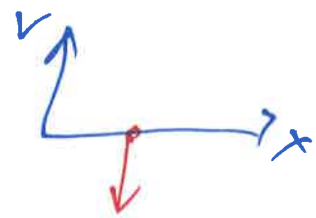
Continuity of $\Psi(x)$

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(x) + V(x)\Psi(x) = E\,\Psi(x)$$

$\Psi(x)$ is continuous for most common potentials

- $V(x)$ is continuous  Ψ', Ψ continuous

- $V(x)$ has discontinuities 
  but finite (e.g. finite square well)
  Ψ'' has discont., [Ψ', Ψ continuous]

- $V(x)$ has discont. and is infinite 
  - (e.g. infinite square well)
    Ψ' is discontinuous, Ψ is continuous
  - (e.g. delta function well) 

- If $V(x)$ contains derivatives of delta functions then $\Psi(x)$ will not be continuous.

Georg Cantor's Diagonalization Proof

There are more real numbers than integers.

Try to List every real number between 0 and 1

.7154925---

.10000000...

.5555555.... -

.2147999279---

⋮

.3142

aleph-null

Cardinality of the set of integers $\equiv \aleph_0$

" " real $\equiv C$

$C = \aleph_1$ continuum hypotheses

Zermelo-Frankel set theory + Axiom of Choice. $\boxed{ZFC}$

Fourier Series – denumerably infinite number of ω's

" transform – non-denumerably " " ω's

$|\psi\rangle$ ket contains everything that can be known about the quantum system

$\langle x|\psi\rangle \equiv \psi(x)$ old friend the wavefunction

The ket $|\psi\rangle$ expressed in the $|x\rangle$ basis

$|x\rangle$ is an eigenstate of the $\hat{x}$ operator

eigenvalue, real number

$$\underset{\text{op}}{\hat{x}}\,\underset{\text{ket}}{|x_0\rangle} = x_0 \underset{\text{ket}}{|x_0\rangle}, \quad \langle x|x_0\rangle = \delta(x-x_0)$$

$|p\rangle$ is an eigenstate of the $\hat{p}$ momentum op.

eigenvalue, real number

$$\underset{\text{op}}{\hat{p}}\,\underset{\text{ket}}{|p_0\rangle} = p_0 \underset{\text{ket}}{|p_0\rangle} \qquad \langle p|q\rangle = \delta(p-q)$$

$\langle p|\psi\rangle \equiv \tilde{\psi}(p)$ Fourier transform of $\psi(x)$

the ket $|\psi\rangle$ expressed in the $|p\rangle$ basis.

$\langle x|p\rangle = ?$ $\quad |p\rangle$ momentum eigenstate $\Rightarrow$ plane wave

$$= \frac{1}{\sqrt{2\pi\hbar}} e^{\frac{ipx}{\hbar}}$$

$$\langle p|x\rangle = \langle x|p\rangle^* = \frac{1}{\sqrt{2\pi\hbar}}\, e^{-\frac{ipx}{\hbar}}$$

$$\tilde{\psi}(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} dx\, \psi(x)\, e^{-\frac{ipx}{\hbar}}$$

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} dp\, \tilde{\psi}(p)\, e^{+\frac{ipx}{\hbar}}$$