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The concept of the photon

It has its logical foundation in the quantum theory of radiation. But the "fuzzy-ball" picture of a photon often leads to unnecessary difficulties.

Marlan O. Scully and Murray Sargent III

The idea of the photon has stirred the imaginations of physicists ever since 1905 when Einstein originally proposed the use of light quanta to explain the photoelectric effect. This concept is formalized in the quantum theory of radiation, which has had unfailing success in explaining the interaction of electromagnetic radiation with matter, seemingly limited only by the ability of physicists to perform the indicated calculations. Nevertheless, it has its conceptual problems—various infinities and frequent misinterpretations. Consequently an increasing number of workers are asking, "to what extent is the quantized field really necessary and useful?" In fact the experimental results of the photoelectric effect were explained by G. Wentzel in 1927 without the quantum theory of radiation. Similarly most electro-optic phenomena such as stimulated emission, reaction of the emitted field on the emitting atom, resonance fluorescence, and so on, do not require the quantization of the field for their explanation. As we will see, these processes can all be quantitatively explained and physically understood in terms of the semiclassical theory of the matter-field interaction in which the electric field is treated classically while the atoms obey the laws of quantum mechanics. The quantized field is fundamentally required for accurate descriptions of certain processes involving fluctuations in the electromagnetic field: for example, spontaneous emission, the Lamb shift, the anomalous magnetic moment of the electron, and certain aspects of blackbody radiation. (The Compton effect also fits here, but see later under references 8b and c.) Here we will outline how the photon concept originated and developed, where it is not required and is often misused, and finally where it plays an essential role in the understanding of physical phenomena.[1] In our discussion we will attempt to give a logically consistent definition of the word "photon"—a statement far more necessary than one might think, for so many contradictory uses exist of this elusive beast. In particular consider the original coining of the word by G. N. Lewis:[2]

> "[because it appears to spend] only a minute fraction of its existence as a carrier of radiant energy, while the rest of the time it remains an important structural element within the atom . . . , I therefore take the liberty of proposing for this hypothetical new atom which is not light but plays an essential part in every process of radiation, the name photon!"

(our exclamation point). Clearly the present usage of the word is very different.

From Maxwell to Schrödinger

Although the nature of light has been a subject of wonder since day one[3] (it was on a Monday), the conceptual basis for the understanding of radiative phenomena begins with James Clerk Maxwell and Heinrich Hertz. While it is true that Isaac Newton, Christian Huygens, Thomas Young, and many others contributed mightily to our understanding of optics, the Maxwell–Hertz demonstration that light is made of the same stuff as electric and magnetic fields must be regarded as the first insight into the inner workings of the radiation field. According to their description, light is radiated by accelerating charges and is an electromagnetic excitation (of an "aether").

Among other things, it must have been the far-ranging success of Maxwell in explaining electromagnetic phenomena that led 19th-century physicists to state that there were really only two clouds on the horizon of physics at the beginning of the 20th century. Interestingly enough, both of these clouds involved electromagnetic radiation. The first cloud, namely the null result of the Michelson–Morley experiment, led to special relativity, which is the epitome of classical mechanics, and really capped things off in a logical way. The second cloud, the Rayleigh–Jeans catastrophe and the nature of blackbody radiation, led to the beginnings of quantum mechanics, which, of course, was a radical change in physical thought up to that point. Note that while both of these problems involve the radiation field, neither (initially) involved the concept of a photon. That is, neither Einstein nor Lorentz in the first instance nor Max Planck in the second called upon the particulate nature of light for the explanation of the observed phenomena. Relativity is strictly classical, and Planck only quantized energies of the oscillators in the walls of his cavity, not the field. Up to this point (before 1905) the discreteness of light quanta was never invoked.

The next chapter in the history of the photon concept came when Einstein applied Planck's quantization ideas to the photoelectric effect. The situation here was very different from that envisioned by Planck. Einstein invoked the existence of discrete bundles or

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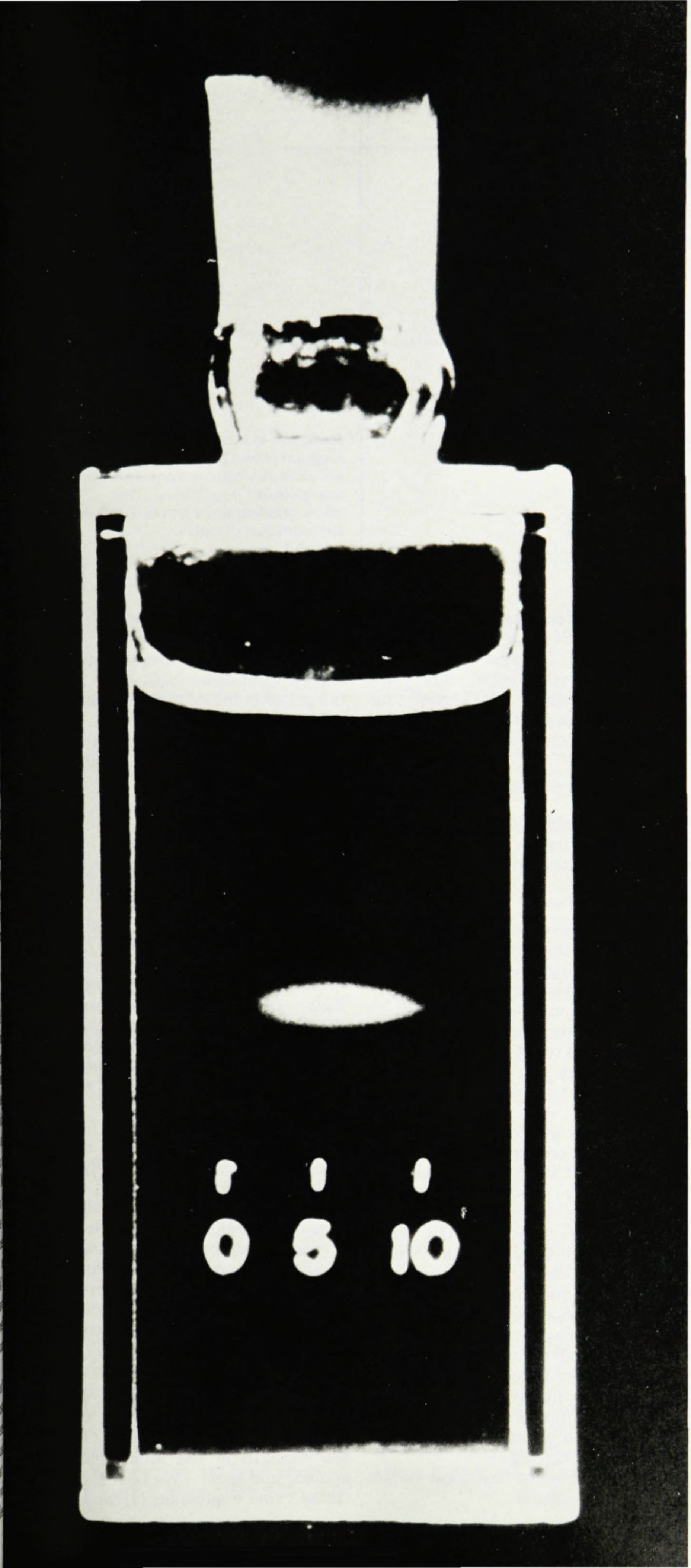
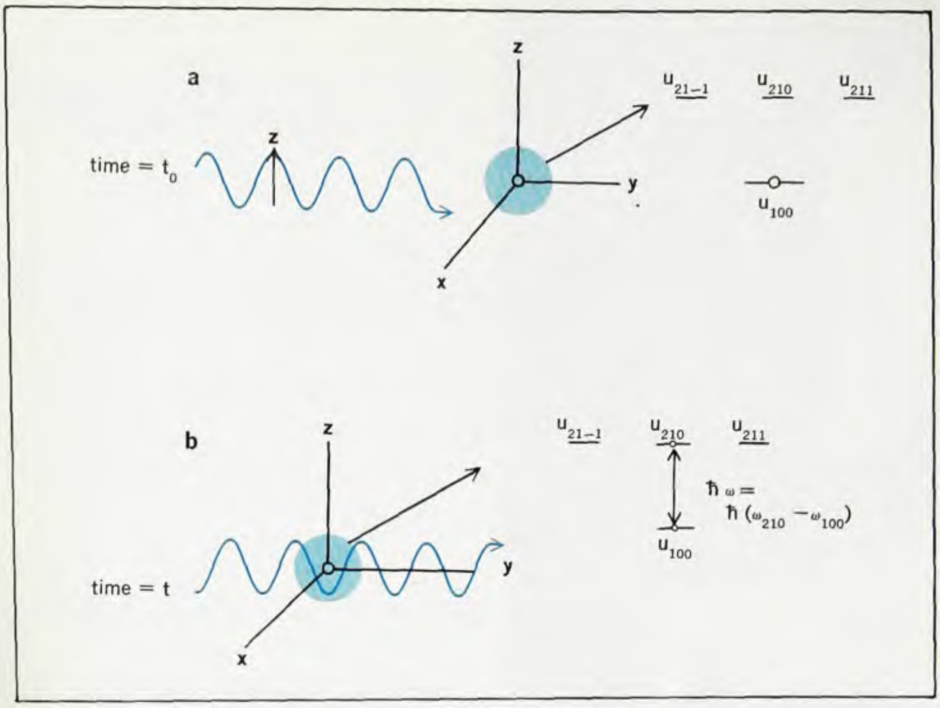


Laser pulse photographed in flight.
This is an ultrashort green pulse obtained as the second harmonic of 1.06-micron light from a neodymium-doped glass laser. To make the photograph the pulse was passed through a water cell; the scale on the cell wall is in millimeters. The camera shutter was a Kerr cell, triggered by an infrared pulse (1.06 micron) from the same laser; exposure time was about 10 picosec, the same period as the duration of these ultrashort pulses. During the exposure the pulse moved about 2.2 mm (right to left), the velocity of light in the cell being approximately 2.2×10^{10} cm/sec. (Photograph by Michel Duguay, Bell Laboratories.)



Radiation-induced transitions. This diagram shows how the electric field of equation (2) induces transitions from the energy levels u_{100} to u_{210}. The corresponding wave function is given by a superposition of states.
Figure 1

quanta of light energy (photons) to explain the ejection of photoelectrons from solids. This was distinct from Planck's idea in which only matter was quantized, as is illustrated in the following passage from George Gamow's delightful little book, *The Thirty Years That Shook Physics*[4]

"Having let the spirit of the quantum out of the bottle, Max Planck was himself scared to death of it and preferred to believe the packages of energy arise not from the properties of the light waves themselves but rather from the internal properties of the atoms which can emit and absorb radiation only in certain discrete quantities. Radiation is like butter, which can be bought or returned to the grocery store only in quarter-pound packages, although the butter as such can exist in any desired amount .... Only five years after the original Planck proposal, the light quantum was established as a physical entity existing independently of the mechanism of its emission or absorption by atoms. This step was taken by Albert Einstein in an article published in 1905, the year of his first article on the Theory of Relativity. Einstein indicated that the existence of light quanta rushing freely through space represents a necessary condition for explaining empirical laws of the photoelectric effect; that is, the emission of electrons from the metallic surfaces irradiated by violet or ultraviolet rays."

We shall return to the photoelectric effect later; however, we note that Planck's "butter-ball quantum" idea is not completely absurd, and, in fact, a modern version of it is being reconsidered by some modern theoretical physicists.[8]

The next cornerstone is the realization that matter itself has a wave-like side to its personality. The first to put this in a concrete mathematical form was Erwin Schrödinger. He wrote his famous equation for the wave function of an atom, $\psi(r,t)$ in terms of its Hamiltonian as

$$i\hbar \frac{\partial}{\partial t} \psi(r,t) = \mathcal{H}\psi(r,t) \qquad (1)$$

and is responsible for demonstrating that the wave nature of matter is essential for its understanding.

Semiclassical theory

Atoms require quantum theory in the description of their behavior, because among other things, classical mechanics tells us that orbiting (therefore accelerating) electrons in atoms should radiate and spiral into the nucleus in contradiction of observed results! A surprisingly successful theory of the atom-field interaction can be obtained in which the atoms obey the laws of quantum mechanics and the electric field is treated *classically* according to Maxwell's equations—that is to say, without the concept of the photon. This semiclassical theory is important for our present purposes for two reasons: First it is important to understand which classical phenomena do not need or logically imply quantized fields for their explanation, and second, the semiclassical theory accounts quantitatively for most radiation-matter interactions. In this part of our article we will support this contention by reviewing the semiclassical description of:

- the response of an atom to a resonant, monochromatic field
- the self-consistent treatment of the atom-field interaction
- stimulated emission
- resonance fluorescence
- the photoelectric effect

Consider first a hydrogenic atom with energy eigenstates u_{nlm}, and suppose the atom is initially in its ground (1s) state, u_{100}, with energy $\hbar\omega_{100}$. We irradiate the atom by a light beam represented by the linearly polarized, plane-wave electric field

$$\mathbf{E}(y,t) = \hat{z}\, E_0 \cos(\nu t - Ky) \qquad (2)$$

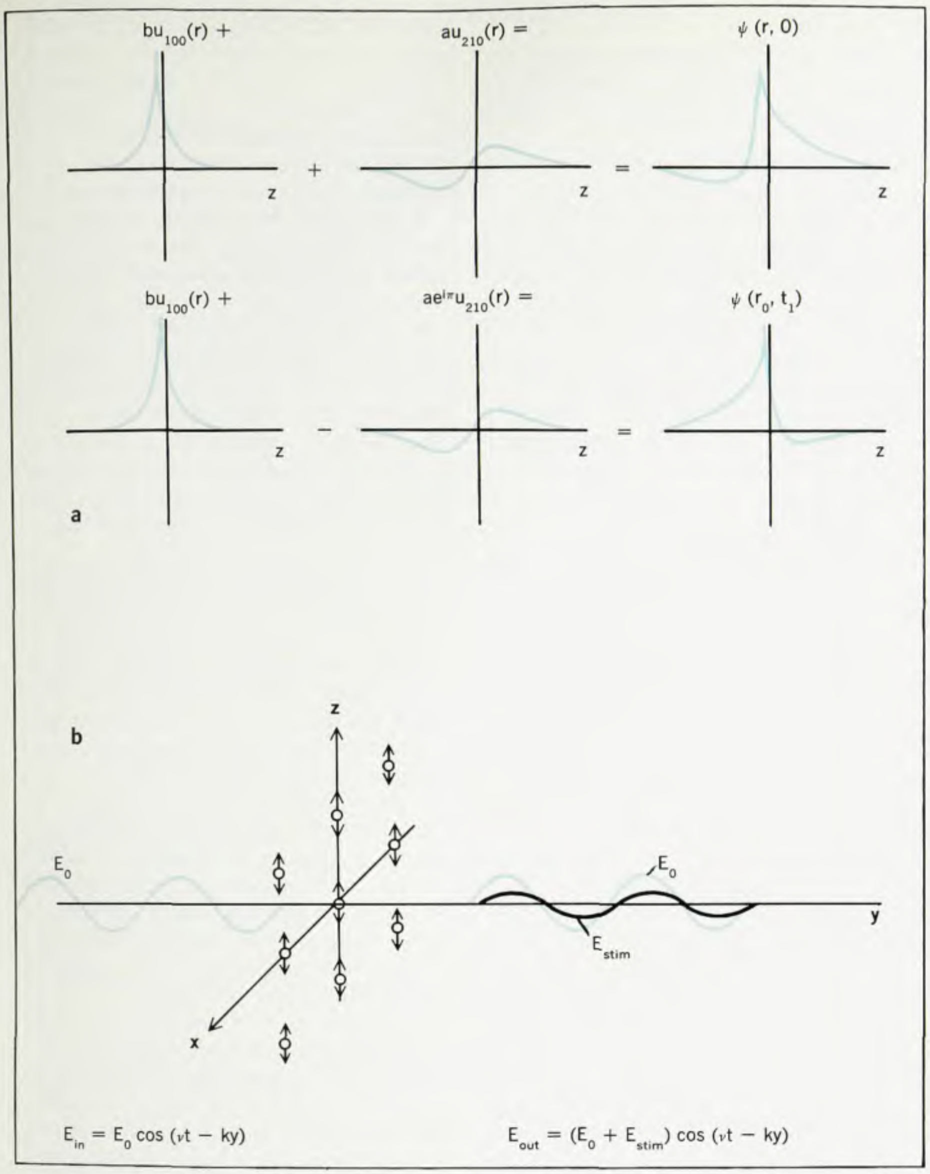
where the (circular) frequency ν is nearly resonant with the 1s → 2p ($u_{100} \to u_{210}$) transitions, that is, $\nu \approx \omega \equiv \omega_{210} - \omega_{100}$. Radiation so polarized induces transitions from the u_{100} level to the u_{210} level, causing the wave function $\psi(\mathbf{r},t)$ to become a linear superposition of the two eigenfunctions as depicted in figure 1. The time development of $\psi(\mathbf{r},t)$ is determined by the Schrödinger equation (1) whose Hamiltonian includes the electric dipole interaction energy

$$\mathcal{H}_1 = -e\mathbf{r}\cdot\mathbf{E} \qquad (3)$$

The resulting z dependence of $\psi(\mathbf{r},t)$ varies in time as shown in figure 2a. There the probability density $\psi^*\psi$ oscillates back and forth across the (positively charged) nucleus with frequency $\omega = \omega_{210} - \omega_{100}$. Hence an ensemble of N such systems located in a volume (small compared to a cubic wavelength) about the position $\mathbf{R}_0$ produces an average oscillating dipole moment, namely

$$\mathbf{p}(\mathbf{R}') = N\left[\int d^3r\psi(\mathbf{r},t)e\mathbf{r}\psi(\mathbf{r},t)\right]\delta(\mathbf{R}' - \mathbf{R}_0) \qquad (4)$$

which depends on the detuning $(\omega - \nu)$, the strength of the atom-field interactions, and so on. We treat the expectation-value expression (4) as an ordi-



Radiating dipoles. The z-dependence of the wave function $\psi(\mathbf{r},t)$ in part (a) are for $t = 0$ and $t = \pi/\omega$; the probability density $\psi^*\psi$ oscillates back and forth across the nucleus at frequency ω, so yielding an oscillating dipole. Part (b) of the figure shows how a sheet of dipoles radiates an electric field in phase with the incident field (equation 2). The individual fields become increasingly retarded the further off the axis one goes.
Figure 2

nary dipole density radiating, for example, the far-field electric field

$$\mathbf{E}(\mathbf{R},t) = [\omega^2/(4\pi\epsilon_0 c^2)]\,(n \times \mathbf{p}) \times \hat{n}\,\frac{\exp[iK|\mathbf{R} - \mathbf{R}_0|]}{|\mathbf{R} - \mathbf{R}_0|} + \text{complex conjugate} \quad (5)$$

We see that the field induces a dipole moment in an ensemble of atoms and that this moment, in turn, contributes a field. So far we have neglected the effect of the *back reaction* of light emitted by the dipole back on itself. This reaction is included by requiring that the field be self consistent, that is, that the field which the atoms see be consistent with the field radiated, as outlined in figure 3. Solving the self-consistent set of equations in figure 3 simultaneously we account for the back action. This effect is one phenomenon sometimes said to require the quantum theory of radiation. We now apply the semiclassical method to several other problems.

Stimulated emission is the first of these. We wish to study a sheet of atoms in the x–z plane (figure 2b) subject to the incident electric field of equation (2). We suppose again that the atoms have two relevant levels, this time having atomic decay phenomena associated with, for example, collisions, and we carry out the appropriate time integrations to find the dipole density as in equation (4). We find the dipole moment density at the point (x, 0,z)

$$\mathbf{P}(x,0,z,t) \propto \hat{z}E_0[\gamma \sin(\nu t) + (\omega - \nu)\cos(\nu t)] \quad (6)$$

in which the constant of proportionality depends on the number of atoms involved, the strength of the atom–field interaction, a Lorentzian involving detuning, the atomic decay rate γ, and so forth. The things to note from equation (6) are:

▶ the dipole oscillates at the driving frequency ν and not the atomic line center ω

▶ the magnitude of the dipole is proportional to the field amplitude E_0

▶ there are components "in phase" ($\cos \nu t$ term) and "in quadrature" ($\sin \nu t$ term) with the inducing field of equation (2). The former modifies the index of refraction in the sheet; the latter acts as a source for gain due to stimulated emission.

Considering the second of these more closely, we note that on resonance the polarization, equation (6), is proportional to $\sin \nu t$. This is 90 deg out of phase with the applied field of equation (2), and in view of equation (5), one notes the radiated field (on axis) is not in phase either. In order to get back into phase with the incident field (indeed in phase with the textbooks!), we add up the contributions from a sheet of dipoles (integrate over x and z) to find a radiated field proportional to $\cos \nu t$ as is equation (2). This second radiated field has the same phase, frequency and direction as the incident field.

Resonance fluorescence[5] is our second application of semiclassical theory and is defined to be the emission of radiation by a ground-state atomic ensemble excited by an optical field. As depicted in figure 4, an incident field (spectral width Γ) and central frequency ν is absorbed by the ensemble (spectral width γ) which, in turn, emits into some new direction with the same spectrum as the incident field if Γ is small compared to γ. This follows from equation (7), which shows that the induced dipole has the same frequency as the inducing (driving) field. Alternatively, for a field whose spectral width is due to its finite duration $1/\Gamma$, we understand the atomic response as that of a driven oscillator with the frequency of the driving field. The atomic oscillator scatters for as long as it is driven, that is, for a time $1/\Gamma$. The spectral width of the emitted radiation therefore corresponds to the reciprocal of the lifetime, namely, Γ.

The photoelectric effect[5,6,8a] is our

The uncertainty principle violated

As an aside, we can note for example that the commutation relations for an atom damped by a quantized field are time independent, because of the presence of quantized Langevin noise sources associated with the atomic damping. These sources are quantized because the field is. Even if we could damp the atom with a classical field, as Michael Crisp and Ed Jaynes suggest, the associated Langevin sources would commute and allow the atomic commutation relations to decay to zero in time. (For further discussion see the paper by Melvin Lax, reference 10.)

This time dependence of the commutation relations then implies a violation of the uncertainty principle. This point was noted first (in a somewhat different way) by Niels Bohr and Leon Rosenfeld.

final example of semiclassical theory. Some readers may find this surprising, because the photoelectric effect provided the original impetus for ascribing a particle character to light. The three main facts of life, photoelectron-wise, are:

▶ When light shines on a photoemissive surface, electrons are ejected with a kinetic energy equal to Planck's constant times the frequency ν of the incident light less some work function ϕ, usually written as

$$\hbar\nu = \frac{mv^2}{2} + \phi \qquad (7)$$

▶ The rate of electron ejection is proportional to the square of the electric field of the incident light (ejection rate $\propto E_0{}^2$),

▶ There is not necessarily a time delay between the instant the field is turned on and the ejection of photoelectrons.

To explain these three characteristics, we suppose the medium consists of ground-state ($|g\rangle$) electrons which can make transitions under the influence of an applied field, equation (2), to a quasi-continuum consisting of momentum states $|k\rangle$ as depicted in figure 5. With the electric-dipole energy, equation (3), and the philosophy of Fermi's Golden Rule, we find the probability for a transition from the ground state to the kth excited state within a time t to be

$$P_k = 2\pi[e|r_{kg}|\hbar]^2E_0{}^2t\delta[\nu - (\epsilon_k - \epsilon_g)/\hbar] \qquad (8)$$

Writing energy $\epsilon_k - \epsilon_g$ as $mv^2/2 + \phi$ as in figure 5, we find that the δ function in equation (8) implies equation (7). This result conflicts with what is often taught, as the following quote from a well known text[7] illustrates:

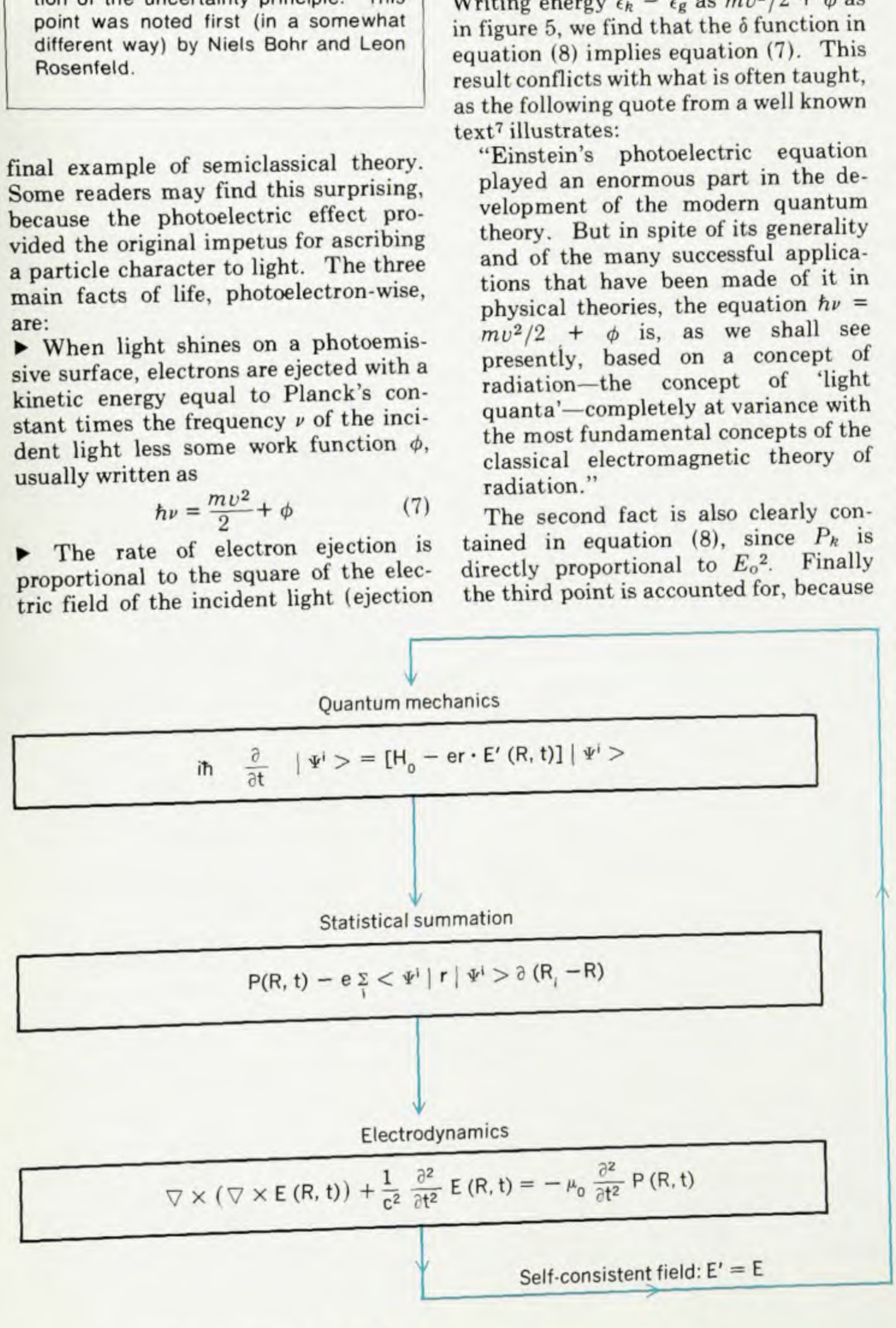
> "Einstein's photoelectric equation played an enormous part in the development of the modern quantum theory. But in spite of its generality and of the many successful applications that have been made of it in physical theories, the equation $\hbar\nu = mv^2/2 + \phi$ is, as we shall see presently, based on a concept of radiation—the concept of 'light quanta'—completely at variance with the most fundamental concepts of the classical electromagnetic theory of radiation."

The second fact is also clearly contained in equation (8), since P_k is directly proportional to $E_0{}^2$. Finally the third point is accounted for, because equation (8) is nonzero even for small times, a fact underlined by Peter A. Franken.[6] "As for the time delays [in the photoelectric effect], quantum mechanics teaches us that the *rate* is established when the perturbation is turned on [after several optical cycles]."

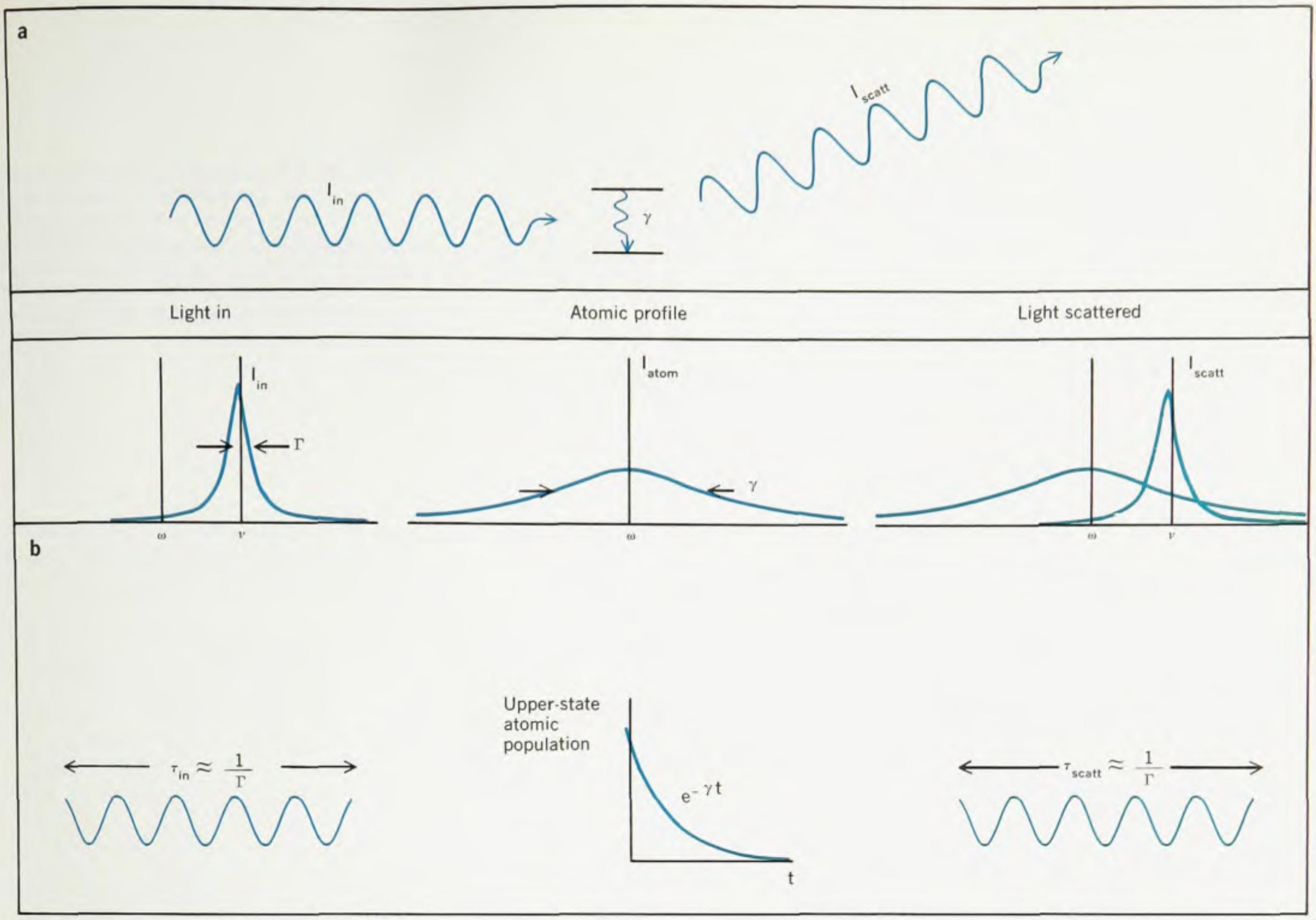
In fact, for the majority of quantum optical calculations the semiclassical theory proves most adequate. We note that in addition to those examples above, nonlinear optics,[7a] much of laser theory,[7b] pulse-propagation phenomena[7c] and even "photon" echo[7d] are all best explained without photons. That the list of successes of semiclassical theory is impressive is further illustrated by the following quote from the recent paper of Michael D. Crisp and Ed T. Jaynes,[8]

> "Even though it is generally believed that a full quantum-electrodynamic treatment is necessary in order to obtain all radiative effects correctly, many calculations involving the interaction of radiation and matter were first done without quantizing the electromagnetic field. Thus is the case of the photoelectric effect,[8a] the scattering of radiation from a free electron (Klein–Nishina formula)[8b] stimulated emission and absorption of radiation by an atom,[8c] and vacuum polarization,[8d] the correct predictions were first obtained by semiclassical methods."

They continue with the assertion that, while spontaneous emission and the Lamb shift are generally conceded to require the quantized field, the self-consistent semiclassical theory does surprisingly well even here. In fact they derive a "Lamb shift" that is order-of-magnitude correct from their semiclassical calculation! However we



Self-consistent equations demonstrating that an assumed field $E'(\mathbf{R},t)$ perturbs the ith atom according to the laws of quantum mechanics and induces an electric dipole expectation value. Values for atoms localized at $\mathbf{R}$ are added to yield macroscopic polarization, $\mathbf{P}(\mathbf{R},t)$. This polarization acts as a source in Maxwell's equations for a field $\mathbf{E}(\mathbf{R},t)$. The loop is completed by the self-consistency requirement that the field assumed, $\mathbf{E}'$, is equal to the field produced, $\mathbf{E}$.
Figure 3



Resonance fluorescence. In part (a) incident light is scattered by an atom with a lifetime $1/\gamma$. In part (b) the incident light has a spectrum centered at frequency ν and corresponds to a wave train of duration $1/\Gamma$. Scattered light has width $1/\Gamma$, is centered at ν and lasts for time $1/\Gamma$; γ is much greater than Γ. Figure 4.

should note that their semiclassical explanation of spontaneous emission runs into conceptual difficulties for the case of atoms excited to a single eigenstate, because the initial atomic dipole **p** of equation (5) vanishes, resulting in infinite lifetimes. One can argue that this excited state is metastable much like a pencil standing on its point—that is, only a small fluctuation is required to get things started.[9] As we shall see, the quantized field readily provides such fluctuations. Furthermore, in the semiclassical theory, the electronic commutation relations[10] are not necessarily preserved in time, and hence the uncertainty principle for the matter can be violated. (See box on opposite page.)

Finally, and most importantly, the quantitative successes[11] of quantum electrodynamics are so impressive that we are virtually compelled to quantize the field as well as the atoms. In view of these facts, we now turn to the photon concept as it is embodied in the quantum theory of radiation.

The quantum theory of radiation

In 1927, P. A. M. Dirac[12] quantized the radiation field in addition to the atom, and the photon concept was for the first time placed on a logical foundation. We outline here the quantum theory of radiation in a form suitable for our purposes. (The present treatment, which uses E and B instead of the vector potential, follows that of reference 2.) For simplicity, we consider a one-dimensional cavity of length L that has perfectly reflecting mirrors. We take the electric and magnetic fields to be polarized in the z and x directions respectively and to be single modes of the cavity, as shown in figure 6. There we see that the electric and magnetic fields act as position and momentum coordinates. The corresponding energy in the cavity is given by the volume integral of the electric and magnetic field densities:

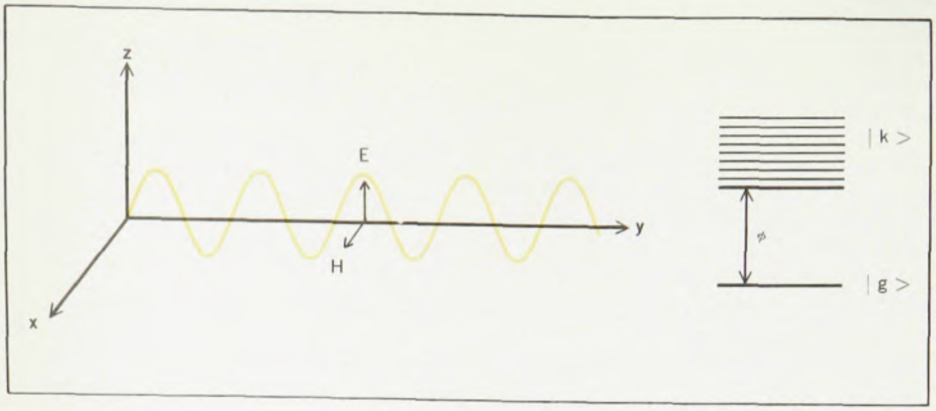
$$\begin{aligned}\mathcal{H} &= \frac{\int [\epsilon_0 E^2 + \mu_0 H^2]}{2}\, d(\text{volume}) \\ &= \frac{p^2 + \Omega^2 q^2}{2}\end{aligned} \tag{9}$$

which is just the energy of a simple harmonic oscillator for a particle oscillating with frequency Ω, mass M and spring constant $M\Omega^2$. A more general multimode field is represented by a collection of such oscillators, one for each mode. To quantize the field (that is, to introduce the photon or particle nature of the radiation) we treat the electric-field "position" coordinate q and the magnetic-field "momentum" p according to the laws of quantum mechanics. We require the commutation relations $[q,p] = i\hbar$; $[q,q] = [p,p] = 0$.

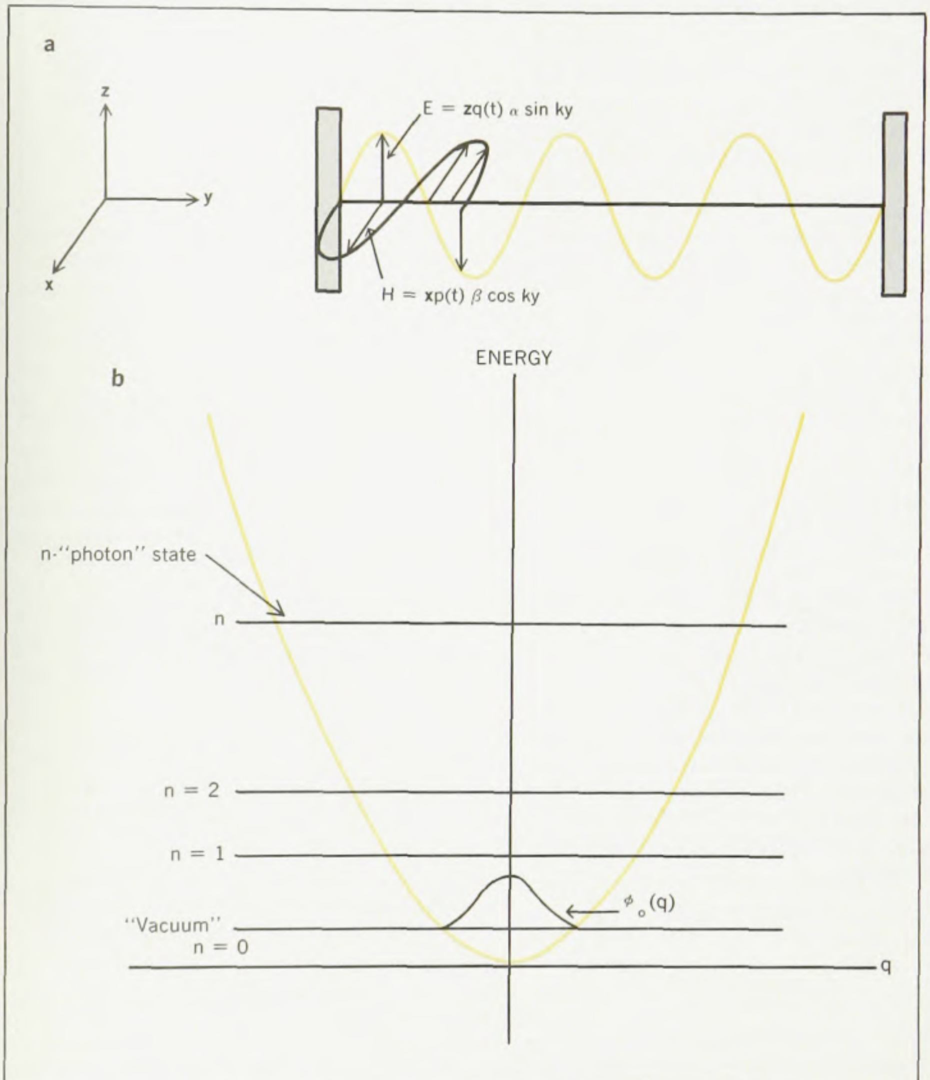
Our single-mode field is then described by the quantum-mechanical wave function

$$\psi(q,t) = \sum_{n=0}^{\infty} c_n(t)\, \phi_n(q) \tag{10}$$

where $|c_n|^2$ is the probability that the radiation oscillator is excited to the nth-energy eigenstate characterized by the eigenfunction $\phi_n(q)$ (the usual Hermite polynomial multiplied by a Gaussian) and having energy $\hbar\Omega(n + 1/2)$. This n-quantum state is said to be the "n-photon" state; that is, $\phi_0(q)$ has no photons (the vacuum), $\phi_1(q)$ has one photon, and so on. We note that the introduction of the wave function $\psi(q,t)$ (Schrödinger picture) or equivalently the noncommutativity of the operators p and q (Heisenberg picture) has the



Photoelectric effect. An incident electromagnetic field interacts with a system in its ground state $|g\rangle$, causing transitions to occur to excited states $|k\rangle$, that is, ejecting an electron.
Figure 5.



Electric and magnetic fields in a one-dimensional cavity, polarized in the z and x directions respectively. Part (a) shows single-mode standing waves proportional to coordinates q and p, with constants of proportionality α and β respectively. In part (b) we see the simple harmonic oscillator energy-level diagram resulting from quantization of the field. The nth level of the quantized oscillator, $\phi_n(q)$, corresponds to the state having n "photons," while the vacuum is associated with $\phi_0(q)$.
Figure 6

effect of bringing out the *wave side* of "particles" (say, electrons) and the *particle side* of "waves" (say, electric waves).

The q and p operators serve to show that the single-mode electromagnetic field is dynamically equivalent to a simple harmonic oscillator. A more convenient and physically revealing set of operators is the annihilation operator $a \propto \Omega q + ip$ and its adjoint $a\dagger$, the creation operator. As their names suggest, these operators annihilate and create photons when acting on photon number states; in other words, $a(a\dagger)$ lowers (raises) ϕ_n to ϕ_{n-1} (ϕ_{n+1}). They are not Hermitian and hence do not themselves represent observables. However, the electric field is given by the Hermitian combination

$$E(y) = \mathcal{E}\,(a + a^+) \sin (Ky) \quad (11)$$

where $\mathcal{E}$ is the electric field "per photon" and the Hamiltonian is

$$\begin{aligned} \mathcal{H} &= \hbar\Omega\{a^+a + [a,a^+]/2\} \quad (12) \\ &= \hbar\Omega(a^+a + 1/2) \end{aligned}$$

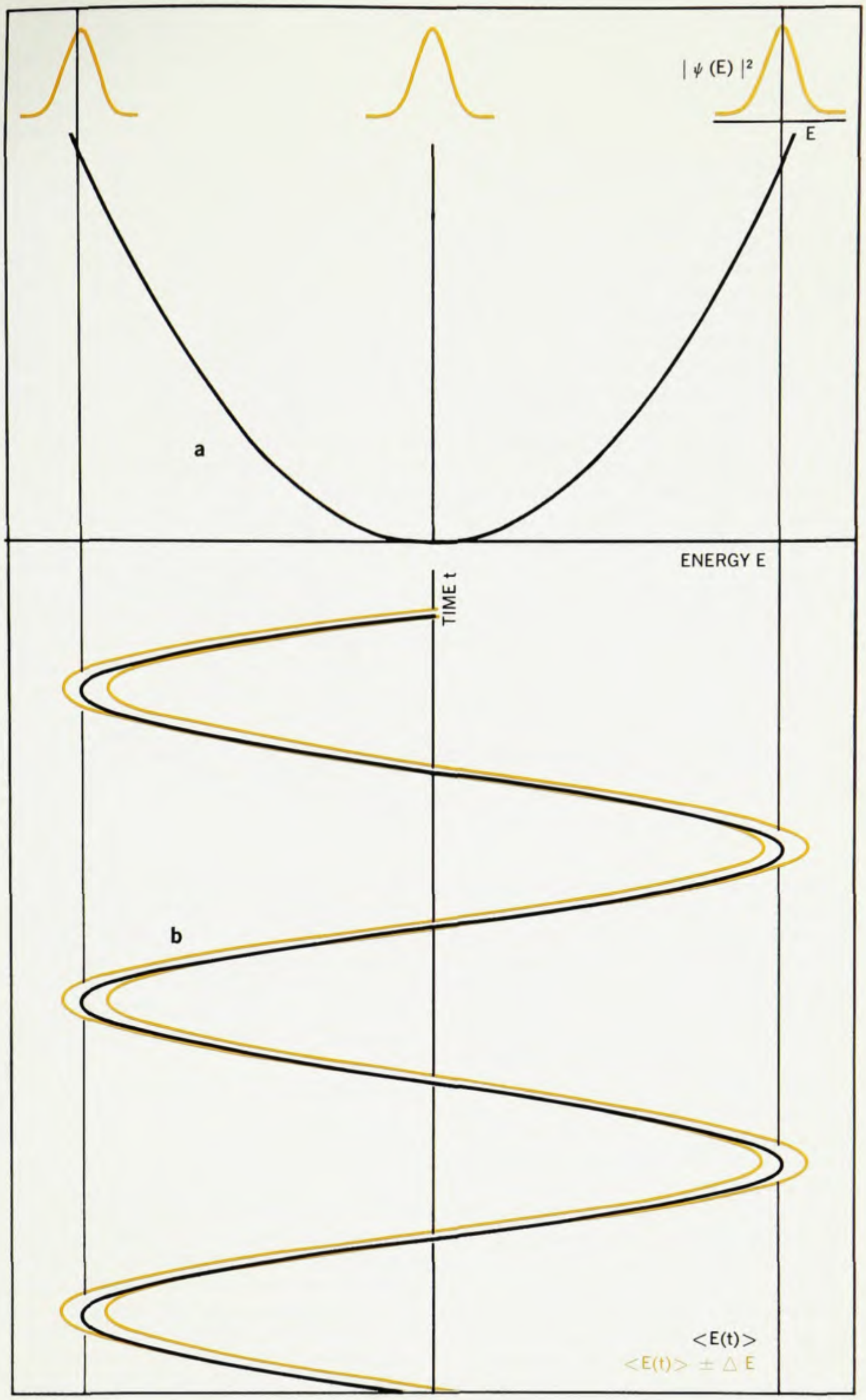
We emphasize that it is the introduction of the *commutation relations* $[q,p] = i\hbar$, or equivalently $[a,a^+] = 1$, that leads to the *photon concept*.

The first thing to note about the quantized field is that it has fluctuations, *even in the absence of "photons."* In fact, denoting the vacuum state (0 photons) by $|0\rangle$, we find the Hamiltonian, equation (12), has the "zero-point" expectation value $\langle 0|\mathcal{H}|0\rangle = \hbar\Omega/2$, the electric field of equation (11) has vanishing expectation value, but that the vacuum average of the field squared is

$$\langle 0|E^2|0\rangle = \mathcal{E}^{\,2} \sin^2 (Ky) \quad (13)$$

Thus the field has fluctuations about a vanishing mean in the vacuum. The zero-point energy $\hbar\Omega/2$ is given by a volume integral of $\langle 0|E^2|0\rangle$ and is therefore called the "energy" of the vacuum fluctuations. We shall outline the success of these considerations in

Time evolution of the expectation value $\langle E \rangle$ of the electric field operator, and variance ΔE indicated by error bars associated with the minimum uncertainty wave packet, are shown in part (a). Part (b) shows the time evolution of a wave packet with minimum uncertainties ΔE and ΔH.
Figure 7



accounting for the statistical fluctuations of light quanta, spontaneous emission, the Lamb shift, and so forth, in a few paragraphs below. However, let us first find out how we regain the classical field, equation (2), from the quantized (photon) field corresponding to the appropriate state vector in equation (10).

We often hear that large quantum numbers correspond to the classical limit. This is a misleading point of the view here, for the expectation value of the field in an n-photon state $|n\rangle$ vanishes, and this fact is true even for $n \to \infty$. The actual classical limit consists of a superposition of photon states, and this fact naturally leads us to a discussion of photon statistics. Essentially we desire a state of the field $|\psi(t)\rangle$ that yields the classical field of equation (2) for the expectation value $\langle E \rangle$, the square of equation (2) for $\langle E^2 \rangle$, and so on—that is, a field with precise amplitude and phase. But we must recall that the electric and magnetic fields correspond to position and momentum, which obey the uncertainty principle, so that

$$\Delta E \Delta H \geq \hbar/2 \times (\text{constant}) \quad (14)$$

The best we can do is to take the minimum uncertainty case (for all time) for which equality in equation (14) holds. This is described by the coherent (particle) packet[13]

$$|\alpha\rangle = \exp(-\alpha^*\alpha/2) \sum_{n=0}^{\infty} (\alpha^n/\sqrt{n!})|n\rangle \quad (15)$$

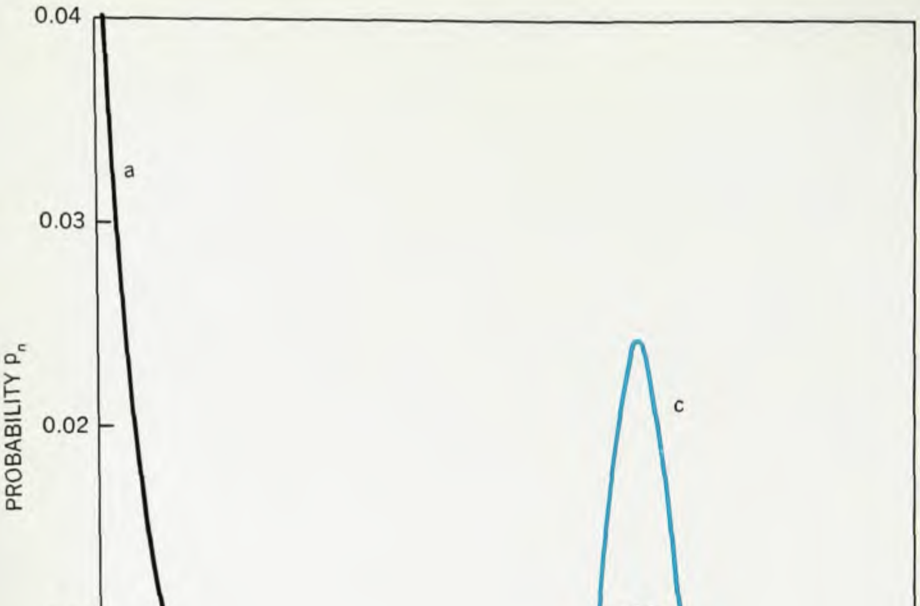
This state is the eigenstate of the annihilation operator a with eigenvalue α. (If statements such as this turn the reader off, we invite him to annihilate them from his copy.) We see in figure 7 that the probability density for this state, $|\langle q|\alpha\rangle|^2$, oscillates back and forth in the harmonic oscillator well without change in shape; that is, it coheres. The amplitude of the classical field, equation (2), is related to the complex constant α and the electric field "per photon" $\mathcal{E}$ by $E_0/2 = \mathcal{E}\,|\alpha|$.

We see that this "most classical" state is *not* a single-photon number state, but rather a superposition with the Poisson probability of having n photons given by

$$P_n = \exp(-\alpha\alpha^*)(\alpha\alpha^*)^n/n! \quad (16)$$

The average photon number $\langle n \rangle$ is thus $|\alpha|^2$, from which we see that the intensity $(\mathcal{E}\,|\alpha|)^2 \propto \langle n \rangle \hbar\Omega$. Equation (16) defines the photon statistical distribution for the coherent state. It is interesting to compare it with that for thermal radiation and that for a laser[14] as shown in figure 8.

It is perhaps worthwile to note that the distinction between the thermal and coherent distributions is by no means merely academic, for the thermal implies a Hanbury-Brown–Twiss correlation, while the coherent does not. This



Photon statistical distributions compared for filtered blackbody (part a), laser (part b) and purely coherent (part c) light beams.
Figure 8

correlation is the measure of the excess probability of finding double photoelectron emission over that given by a purely random sequence of events such as raindrops on a roof. In fact, the probability of double photoelectron excitation is twice as large for the "single-frequency" blackbody distribution as for the purely coherent, Poisson, distribution. It is for this reason that photoelectrons produced by a purely coherent light beam are said to be completely uncorrelated—there is no bunching. In general a light beam is completely characterized not just by its spectral density, but by all higher order correlations as well.

For many problems of interest, it is convenient to expand the radiation state (or density operator) in terms of the coherent states $|\alpha\rangle$ instead of the number states. This is accomplished[13] in terms of the $P(\alpha)$ "distribution" defined by the density-operator expansion

$$\rho = \int d^2\alpha\, P(\alpha)\, |\alpha\rangle\langle\alpha| \qquad (17)$$

where integration is carried out over the complex α plane. For the coherent state $|\alpha_0\rangle$, $P(\alpha) = \delta^2(\alpha - \alpha_0)$. For "single-mode" thermal radiation

$$P(\alpha) = \exp\left(-|\alpha|^2/\langle n\rangle\right) \qquad (18)$$

$P(\alpha)$ gives one a measure of deviation from the classical state.

We now have at our disposal a simple method to show the validity of Einstein's[15] correct (but not immediately accepted[16]) interpretation of the fluctuations in the blackbody spectrum. He claimed that, although the Planck law could be accounted for with a classical field (as Planck originally derived it), the energy fluctuations $(\Delta\mathcal{H})^2$ contained a part due to the wave nature of light plus a part due to its particle character. His formula is (in our notation)

$$(\Delta\mathcal{H})^2 = \bar{\mathcal{H}}\hbar\Omega + \mathcal{H}^2/g(\Omega) \qquad (19)$$

where the average energy

$$\bar{\mathcal{H}} = \hbar\Omega\langle n\rangle g(\Omega) \qquad (20)$$

and $g(\Omega)$ is the density-of-states factor. He identified the first term in equation (19) with the particle character and the second with the wave. Not everyone agreed. In fact, using the information then (1913) available, Wilhelm Wien[16] argued that there was no special reason to attribute the fluctuations to two separate causes (particle and wave).

We see that Einstein was indeed correct, by using the $P(\alpha)$ distribution of equation (18), to calculate the required averages as the following two-line derivation indicates. We note that $\langle\mathcal{H}\rangle = \hbar\Omega\langle n\rangle$ (dropping the $\hbar\Omega/2$ which ultimately cancels equation (19)) and find

$$\begin{aligned}\langle\mathcal{H}^2\rangle &= (\hbar\Omega)^2 \int d^2\alpha \exp\left(-|\alpha|^2/\langle n\rangle\right) \times \\ &\qquad \langle\alpha|a^\dagger a a^\dagger a|\alpha\rangle \\ &= (\hbar\Omega)^2 \int d^2\alpha \exp\left(-|\alpha|^2/\langle n\rangle\right) \times \\ &\qquad \langle\alpha|a^\dagger a[a,a^\dagger] + a^\dagger a^\dagger aa|\alpha\rangle \\ &= \langle n\rangle(\hbar\Omega)^2 + 2(\hbar\Omega)^2\langle n\rangle^2 \qquad (21)\end{aligned}$$

in which the first term resulted from the commutation relation $[a,a^\dagger] = 1$, that is, from the quantum character of the field, and the second from the wave character of a classical average over intensity. Calculating the mean-square-deviation density, $(\Delta\mathcal{H})^2 = [\langle\mathcal{H}^2\rangle - \langle\mathcal{H}\rangle^2]\, g(\Omega)$, we find Einstein's formula, equation (19). We see that he correctly identified the particle and wave contributions, a noteworthy feat and a tribute to his insight inasmuch as the quantum theory of radiation was not developed until twenty years later!

As mentioned in our semiclassical discussion, the quantum theory of radiation accounts neatly for spontaneous emission.[17] To see this, we use the electric-field operator, equation (11), in the electric-dipole perturbation energy of equation (4). Using the Fermi Golden Rule we find the spontaneous transition rate (inverse of atomic lifetime)

$$\gamma = (er_{ab})^2\Omega^3/(\hbar\pi c^3) \qquad (22)$$

The emitted radiation is not perfectly monochromatic, for the exponential decay implied by equation (22) yields a Lorentzian frequency profile with width 2γ. We note that the sum over final states of the absolute value squared of equation (21), which enters the Golden Rule, is, in fact, proportional to the vacuum expectation value of E^2; that is, the vacuum fluctuations "stimulate" the atom to emit spontaneously.

Perhaps the greatest triumph of the photon concept is the explanation of the Lamb shift[18] between, for example, the $2s_{1/2}$ and $2p_{1/2}$ levels in a hydrogenic atom. According to the relativistic Dirac theory these levels have the same energy, in contradiction of the experimentally observed frequency splitting of 1057.8 MHz. We can understand the shift intuitively[19] by picturing the electron forced to fluctuate about its "Dirac" position because of the fluctuating vacuum field. Its average displacement $\langle\Delta\mathbf{r}\rangle$, is zero, but the squared displacement has a small positive value from this mean position, $\langle(\Delta\mathbf{r})^2\rangle$. This deviation may change the potential energy the electron experiences in the Coulomb field of the nucleus. To determine how much, we expand the energy in a second-order Taylor series. Noting that the first-order term vanishes in the average over fluctuations, that the fluctuations are isotropic, and that $\nabla^2(1/r) = \delta(\mathbf{r})$, we find the energy shift

$$\begin{aligned}\langle\Delta V\rangle_{\rm fluct} &= \langle V(\mathbf{r}+\Delta\mathbf{r}) - V(\mathbf{r})\rangle_{\rm fluct}\\ &= (1/2)\nabla^2(V)(1/3)\langle(\Delta\mathbf{r})^2\rangle_{\rm fluct}\\ &= (1/6)e^2\delta(\mathbf{r})\langle(\Delta\mathbf{r})^2\rangle_{\rm fluct} \qquad (23)\end{aligned}$$

We may now calculate the shift of the energy eigenvalues $\hbar\omega_{nlm}$ by calculating the matrix element $\int d^3r u_{nlm}^* \langle\Delta V\rangle_{\rm fluct} u_{nlm}$. Because only s states have nonzero probabilities for being at $\mathbf{r} = 0$, only these states are shifted (in this approximation). Computation of $\langle(\Delta\mathbf{r})^2\rangle$ requires more discussion, and we refer the reader again to the texts for a complete discussion. However we emphasize that $\langle(\Delta\mathbf{r})^2\rangle$ is nonzero only because of the $(1/2)[a,a^\dagger]\hbar\Omega$ vacuum fluctuations and is a direct consequence of the quantized field; that is $[a,a^\dagger] \neq 0$. The changes in potential energy account for 1040 MHz of the 1057.8-MHz shift observed between the $2s_{1/2}$ and $2p_{1/2}$ states in atomic hydrogen. When various relativistic corrections and infinities are taken care of, the theory agrees beautifully with the experimental results and provides an impressive confirmation of the quantum theory of radiation.

The anomalous magnetic moment of the electron[20] is more difficult to interpret in simple physical terms because the origin of the spin is buried in the relativistic theory of the electron. Nevertheless, the origin is due to the modification of circulating electronic currents (and hence the magnetic moment), as they are affected by the fluctuating electromagnetic vacuum fields and vacuum polarization.

We conclude with a couple of remarks concerning the so-called wave–particle duality and its effect on interference phenomena. From Newton to Huygens to the present, this point has fascinated and often confounded scientists. Even such an outstanding optics text as Arnold Sommerfeld's[21] contains opaque remarks in this regard such as ". . . the photon theory, at leats in its present state of development, is unable to account precisely for polarization and interference phenomena." Indeed, the "photon theory," as embodied in the quantum theory of radiation, does very well even in these cases. For it is the normal-mode functions $U_k(\mathbf{r})$ that describe interference phenomena in terms of nodal (dark) and antinodal (bright) regions of space. These functions are the same for both classicial and quantum fields. Hence there is no need to switch from quantum to classical descriptions or to introduce a mysterious wave–particle dualism in order to explain interference and diffraction. This point is made clear in Fermi's article[22] on the quantum theory of radiation. He does a Lippman-fringe calculation in which light is emitted from one atom, strikes a mirror perpendicular to its direction of propagation, and is absorbed by a second atom. The calculation shows that the probability of excitation of the second atom varies periodically with its distance from the mirror because of interference between the incoming and outgoing light. Fermi comments that "We may conclude that the results of the quantum theory of radiation describe this phenomenon in exactly the same way as the classical theory of interference."

Recent interference experiments[23,24] involving independent light beams have been made possible by the availability of coherent laser sources. These measurements were largely stimulated by Dirac's comment,[25] "Each photon then interferes only with itself. Interference between two different photons never occurs." The fact that interference between independent lasers *is* observed is not puzzling if we recall that the fringes are described by the normal modes of the system. Dirac's comment is consistent with this experiment in view of the fact that the photon is a quantized excitation of the normal modes of the entire system.

In conclusion: The photon concept as contained in the quantum theory of radiation provides the basis for explaining all known electromagnetic phenomena. However, the "fuzzy-ball" picture of a photon often leads to unnecessary confusion. Finally, most quantum and electro-optical physics is well understood and quantitatively explained semiclassically.

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