Johann Balmer Series

only 4 visible spectral lines in hydrogen

$$\frac{1}{\lambda} = R_y\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$n = 3 \to 2$	red	6563 Å
$4 \to 2$	cyan	4861 Å
$5 \to 2$	blue	4341 Å
$6 \to 2$	violet	4102 Å
$7 \to 2$	UV	

$$QM \to R_{y_\infty} = \frac{m_e e^4}{8c\,\epsilon_0^2 h^3}$$

R_{y_H} replace $m_e \to \mu = \dfrac{m_e m_p}{m_e + m_p}$

Starting n level			
1	Lyman	$2 \to 1$, $3 \to 1$, $4 \to 1$ L_α, L_β, L_γ	All UV
2	Balmer	$3 \to 2$, $4 \to 2$ H_α, H_β	$H_\alpha - H_\delta$ are visible rest are UV
3	Paschen	$4 \to 3$ P_α	Infrared
4	Brackett		Infrared
5	Pfund		Infrared
6	Humphreys		Infrared
7	Hansen-Strong		Infrared

Hydrogen-like systems

ions: He^+, Li^{++}, etc. (one electron)

replace one e by Ze where Z is atomic number
protons e.g. He $Z=2$

replace m_e by appropriate $\mu = \frac{m_e m_\alpha}{m_e + m_\alpha}$

Positronium (e^+e^-) bound state

replace $m_e \rightarrow \mu = \frac{m_e m_e}{m_e + m_e} = \frac{1}{2} m_e$

$Z=1$

Muonium Mu $(\mu^+ e^-)$ form compounds
life 2.2 μs MuCl

$Z=1$

$\mu = \frac{m_e m_\mu}{m_e + m_\mu}$

---

true Muonium: $(\mu^+ \mu^-)$ bound state, not yet observed.

Muonic Hydrogen $(p^+ \mu^-)$ muon-catalyzed fusion

μ reduced mass

$m_e * 196 \rightarrow a^* = \frac{a_0}{196}$

p^+ radius puzzle.

Hydrogen

Bound States $\langle \psi_{n'\ell'm'} | \psi_{n\ell m} \rangle = \delta_{nn'} \delta_{\ell\ell'} \delta_{mm'}$

Scattering States $\langle \psi_{\alpha_1} | \psi_{\alpha_2} \rangle = \delta(\alpha_1 - \alpha_2)$

$$\langle \psi_{\substack{\text{bound} \\ n\ell m}} | \psi_{\substack{\text{scattering} \\ \alpha}} \rangle \equiv \emptyset$$

Symmetry : $V(r)$ potential is spherically symmetric

expect Energy levels do not depend on $\underline{m}$

there are $2\ell+1$ values of m

$\Rightarrow$ energy levels $(2\ell+1)$ degenerate

actually E_n is n^2-degenerate & E_n does not depend on ℓ. } accidental degeneracy

expect symmetry group of Hydrogen to be $SO(3)$ ↓ 3 generators

but in fact group is $SO(4)$ ↑ 6 generators

Kepler

$\vec{A}$ Laplace-Runge-Lenz vector

Correction to Hydrogen Problem

Fine Structure

- Relativisitic correction to Kinetic energy

non-rel: $KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

rel: $KE = \sqrt{(mc^2)^2 + (pc)^2} - mc^2 = (\gamma - 1)mc^2$

$$KE = mc^2\sqrt{1 + \frac{p^2}{m^2c^2}} - mc^2 \Longrightarrow \text{Binomial}$$

$$KE = \frac{\hat{p}^2}{2m} - \underline{\underline{\frac{\hat{p}^4}{8m^3}}} + \cdots$$

- e^- spin- e^- orbit interaction $\quad \vec{L}\cdot\vec{S}$

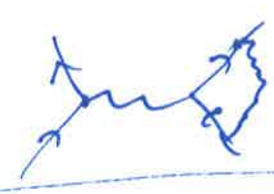
$\alpha \approx \frac{1}{137}$ Fine Structure constant

- Darwin term

Hyperfine correction : Spin Spin interaction
e^- p^+

$\lambda = 21\,cm$ H line

Lamb Shift - Field theory Correction



Bohr	$\alpha^2 m_e c^2$
Fine	$\alpha^4 m_e c^2$
Lamb	$\alpha^5 m_e c^2$ ← Darwin
Hyper-fine	$\left(\frac{m_e}{m_p}\right)\alpha^4 m_e c^2$

Anomalous magnetic moment of e^-

Angular Momentum

Classically $\vec{L} = \vec{r} \times \vec{p} = -\vec{p} \times \vec{r}$

QM: you might worry about the ordering of operators

$$[\hat{X}, \hat{P}_x] = i\hbar \hat{\mathbb{1}} \neq 0 \;,\; [\hat{Y}, \hat{P}_y] \neq 0$$

but $[\hat{X}, \hat{P}_y] = 0$ $[\hat{X}, \hat{P}_z]$ etc.

$$\hat{L}_1 = \hat{L}_x = \hat{Y}\hat{P}_z - \hat{Z}\hat{P}_y$$

$$\hat{L}_2 = \hat{L}_y = \hat{Z}\hat{P}_x - \hat{X}\hat{P}_z$$

$$\hat{L}_3 = \hat{L}_z = \hat{X}\hat{P}_y - \hat{Y}\hat{P}_x$$

QM $\hat{\vec{L}} = \hat{\vec{R}} \times \hat{\vec{P}}$

Commutators

$$[\hat{L}_x, \hat{L}_y] = [(\hat{Y}\hat{P}_z - \hat{Z}\hat{P}_y), (\hat{Z}\hat{P}_x - \hat{X}\hat{P}_z)]$$

$$= \hat{Y}\hat{P}_x \underbrace{[\hat{P}_z, \hat{Z}]}_{-i\hbar} + \hat{X}\hat{P}_y \underbrace{[\hat{Z}, \hat{P}_z]}_{i\hbar}$$

$$= i\hbar(\hat{X}\hat{P}_y - \hat{Y}\hat{P}_x) = i\hbar \hat{L}_z$$

$$[\hat{L}_i, \hat{L}_j] = i\hbar \sum_{k=1}^{3} \epsilon_{ijk} \hat{L}_k$$

recognize Lie algebra $su(2) \simeq so(3)$

Levi-Civita symbol

$$\epsilon_{ijk} = \begin{cases} +1, & ijk \text{ are cyclic eg. } 123,\ 231,\ 312 \\ -1, & \text{odd permutations } 132,\ 213,\ 321 \\ 0, & \text{otherwise, } 112,\ 111 \end{cases}$$

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 = \sum_{i=1}^{3} \hat{L}_i^2$$

$$[\hat{L}^2, \hat{L}_i] = 0 \quad i = 1,2,3 \implies [\hat{L}^2, \hat{\vec{L}}] = 0$$

Can find simultaneous eigenfunctions of $\hat{L}^2$ and $\hat{L}_z$ say.

$$\hat{L}^2 |\psi\rangle = \lambda |\psi\rangle, \quad \hat{L}_z |\psi\rangle = \mu |\psi\rangle$$

Define Ladder operators

$\hat{L}_1^\dagger = \hat{L}_1$ hermitian
$\hat{L}_2^\dagger = \hat{L}_2$ observable

$$\hat{L}_\pm \equiv \hat{L}_1 \pm i \hat{L}_2$$

$$\hat{L}_+^\dagger = \hat{L}_-$$

$$[\hat{L}_3, \hat{L}_\pm] = \pm \hbar \hat{L}_\pm, \quad [\hat{L}^2, \hat{L}_\pm] = 0$$