EPR 1935 Einstein, Podolsky, Rosen

entangled.
2 particles emitted from C
e.g. from a particle decay

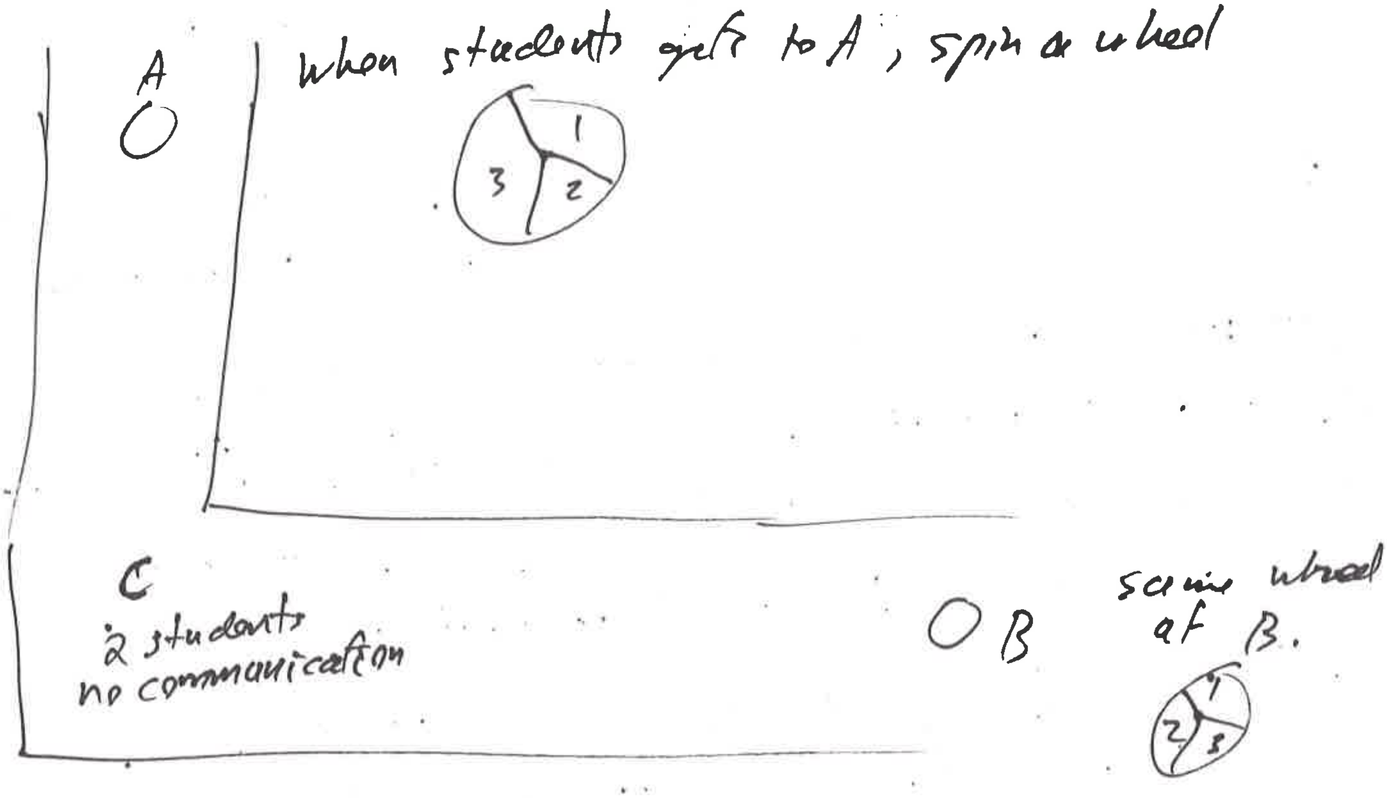
QM → $\hat{x}$ and $\hat{p}$ do not commute, so you can measure precisely $\hat{x}_A$ ($\Delta x_A = 0$) but then you know nothing about $\hat{p}_A$.

EPR → measure $\hat{p}_B$ so far from A that measuring B can not influence A in any way. $P_A = -P_B$, can measure $\hat{x}_A$ ⇒ A must have <u>had</u> a definite x_A, P_A, but they are not both simultaneously measurable in QM.

⇒ QM is incomplete. Can know thing with 100% certainty, but these are not in the theory. (Hidden variables.)

Alternative: Measurements at B influence A, even if 10 billion light year apart. Spooky action at a distance.

"No reasonable definition of reality could be expected to permit this." – EPR



2 Rules

1) If the same number comes up at A and B, then the students must always say the same color. (either red or green).

eg. A1 + B1, A2 + B2, A3 + A3 } both say red or both say green.

If the numbers don't match eg. A1 B2 then students can say anything

A1 B2, could say (red, red), (red green) (green green) (green, red).

2) In a large sample of runs $\frac{1}{2}$ same color, $\frac{1}{2}$ different color.

1) Is easy to achieve: List

L1	L2	L3	...	L8
1 R	1 R	1 G		1 - G
2 R	2 G	2 G		2 G
3 R	3 G	3 R		3 G

Problem: will not ever comply with 2)

e.g. Suppose A and B are given LS
1 R
2 R
3 G

A	B	Colors
1	1	RR
1	2	RR
1	3	RG
2	1	RR
2	2	RR
2	3	RG
3	1	GR
3	2	GR
3	3	GG

match even though numbers are different

Results:
same color $5/9$
different color $\frac{4}{9}$

same results for any list that has two colors. (2 of one, 1 of another).

What about L_1
1 R
2 R
3 R

and L_8
1 G
2 G
3 G

same color – $9/9$
different color – $0/9$

Overall results

same color: $\frac{6}{8}\cdot\frac{5}{9} + \frac{2}{8}\cdot\frac{9}{9} = \frac{2}{3} \approx 67\%$

diff color: $\frac{6}{8}\cdot\frac{4}{9} + \frac{2}{8}\cdot\frac{0}{9} = \frac{1}{3} \approx 33\%$

not 50%-50% rule 2)

This is the essence of Bell's Theorem (Bell's Inequality).
$\Rightarrow$ can't have lists (hidden variables).

One way to do it: QM

Entangle two spin-$\frac{1}{2}$s in a singlet $\psi = \frac{1}{\sqrt{2}}\left[|+-\rangle - |-+\rangle\right]$

total spin $S = S_1 + S_2 = 0$

for A and B

Stern-Gerlach Magnet
A: spin "up" red, "down" green
B: spin "up" green, "down" red.

1) Is satisfied ⇒ If A is "up" along $\hat{z}$ (e.g.) then B is "down" along $\hat{z}$. m_{s_1} and m_{s_2} will be opposite

$m_{s_1} = \pm\frac{1}{2}$, $m_{s_2} = \mp\frac{1}{2}$ always add to 0.

colors will be the same. ✓

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2) Three directions 1, 2, 3 ⇒ $\hat{v}_1, \hat{v}_2, \hat{v}_3$ 120° apart

all equally likely

e.g. $\hat{v}_1 = \cos(0°)\hat{x} + \sin(0°)\hat{y} = \hat{x}$

$\hat{v}_2 = \cos(120°)\hat{x} + \sin(120°)\hat{y} = -\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}$

$\hat{v}_3 = \cos(240°)\hat{x} + \sin(240°)\hat{y} = -\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}$

Colors evenly distributed ⇒ spins average = 0

$$0 \stackrel{?}{=} \sum_{i=1}^{3}\sum_{j=1}^{3} \langle\psi|(\hat{v}_i\cdot\vec{S}_1)(\hat{v}_j\cdot\vec{S}_2)|\psi\rangle$$

pull sums inside the matrix element

$$\sum_{i=1}^{3}\hat{v}_i = 0 = \sum_{j=1}^{3}\hat{v}_j$$

rule 2) ✓

1964 John Stewart Bell.

1980's Alain Aspect at Orsay, France.