Crystals

Bravais Lattice – translational invariance

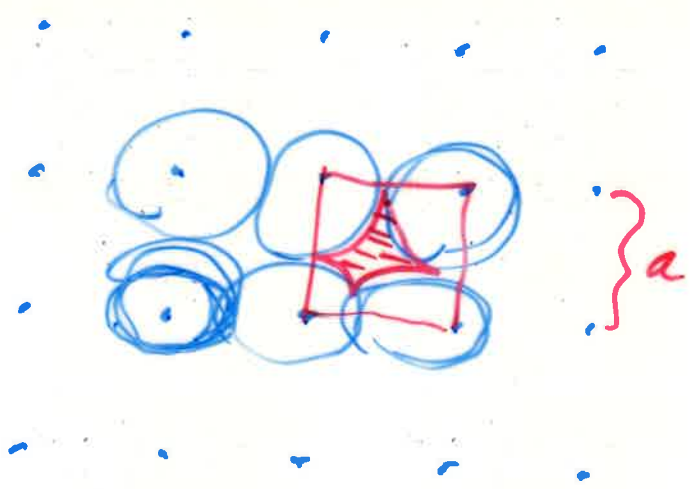
$$\vec{R}' = \vec{R} + n_1\vec{a}_1 + n_2\vec{a}_2 + n_3\vec{a}_3$$

$$n_i = \{\ldots -2, -1, 0, 1, 2 \ldots\}$$

$\{\vec{a}_i\}$ primitive basis vectors.

not necessarily orthogonal. not unique

Eg. square lattice – 2 dimensions 5 total lattices



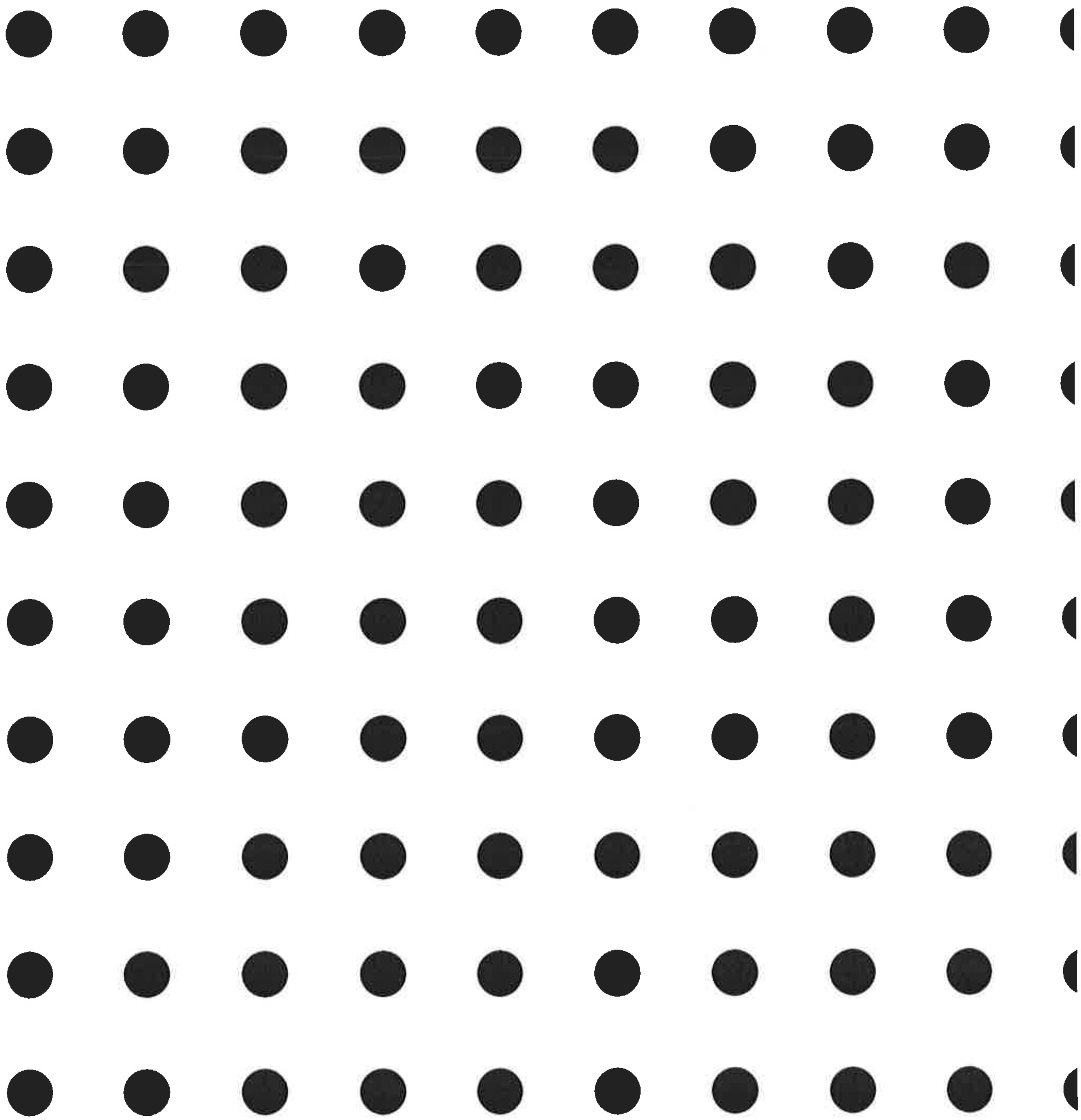
Wigner-Seitz primitive cell (unique)

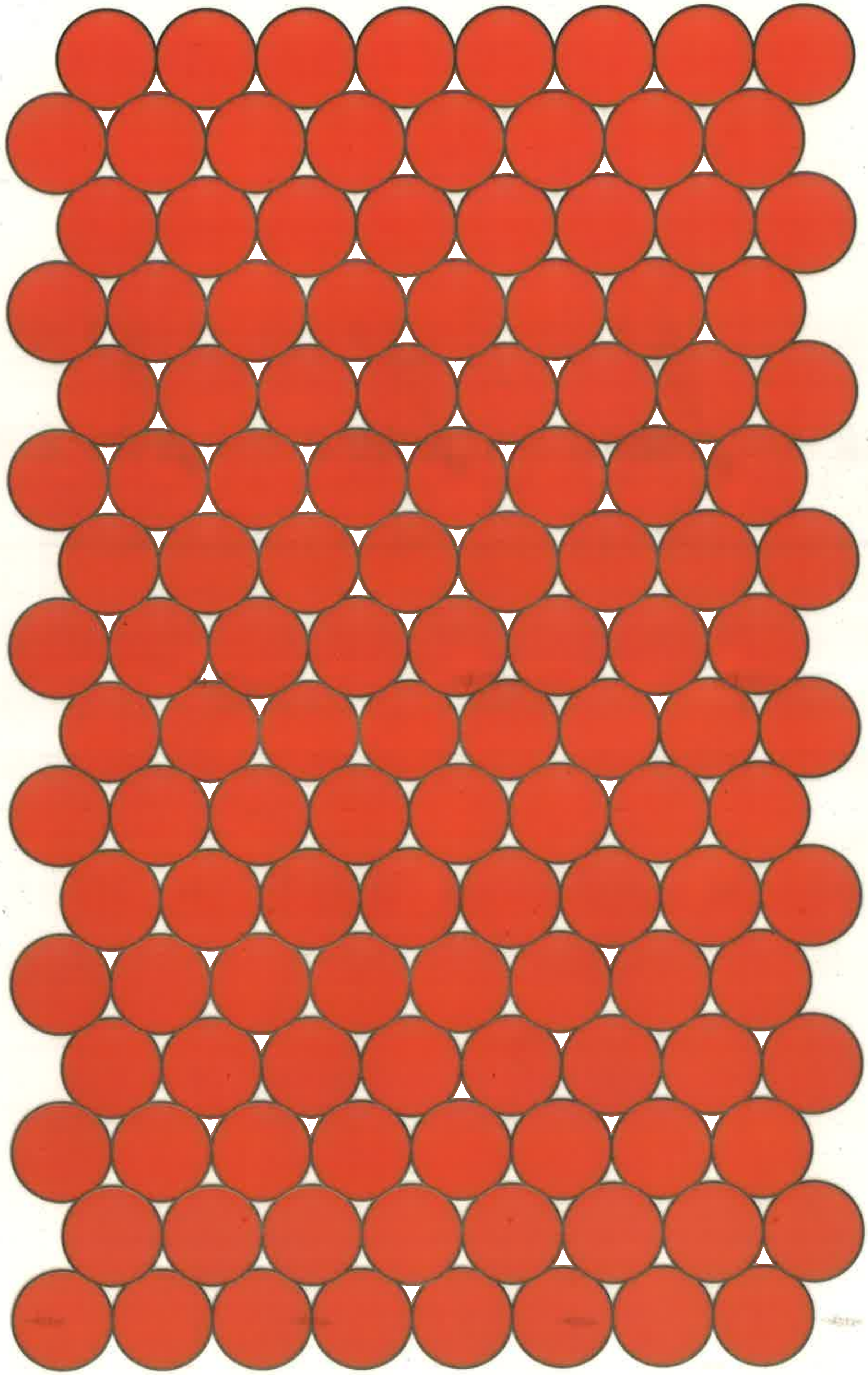
P. cell "volume" a^2

"volume" taken up by the atoms $\frac{\pi a^2}{4}$

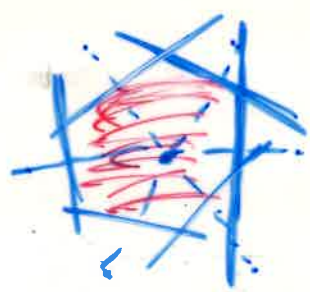
Packing fraction

$$f = \frac{\pi}{4} \approx 0.785$$





hexagonal lattice - 2 dimensions



W-S p-cell
regular hexagon

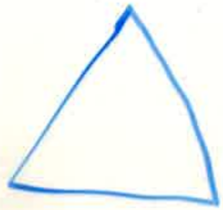
Point Group - Symmetries that carry the lattice into itself, keeping one point fixed.

C_1, C_2, C_3, C_4, C_6 cyclic groups

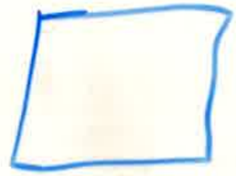
D_1, D_2 D_3 D_4 D_6 dihedral groups

C_1

C_2



D_3



D_4

C_4

D_2

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2 dimensions

(5 Bravais lattices) × (10 points groups)

= ~~50~~ space groups.

17 space groups (Wall paper groups)

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3 dimensions

(14 Bravais lattices) × (32 points groups)

= ~~448~~ space groups

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