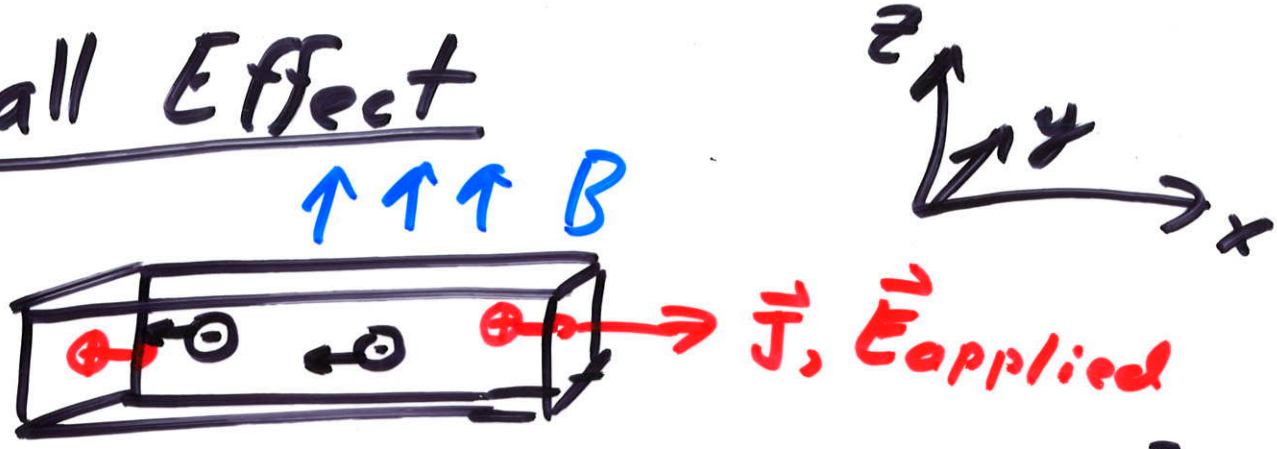
Hall Effect



Ohm's Law $\vec{J} = \sigma \vec{E}$, $\sigma = \frac{n q^2 \tau}{m}$

Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

+++ front --- back

$\vec{E}_{Hall}$ into page

--- front +++ back

$\vec{E}_{Hall}$ out of page

$\vec{F} \neq m\vec{a}$. $\vec{F} = \frac{m\vec{v}}{\tau}$

x) $\frac{m v_x}{\tau} = q(E_x + B_z v_y)$

y) $\frac{m v_y}{\tau} = q(E_y - B_z v_x)$

z) $\frac{m v_z}{\tau} = q E_z$

$E_{Hall} = E_y$ builds up until no more charge can flow in the y-direction

$\Rightarrow v_y = 0$ eventually

$\Rightarrow E_y = B v_x = B \frac{I E_x \tau}{m}$

Hall Coefficient

$$R_H \equiv \frac{E_y}{J_x B_z} = \frac{1}{nq}$$

$R_H < 0$ electrons

$R_H > 0$ holes

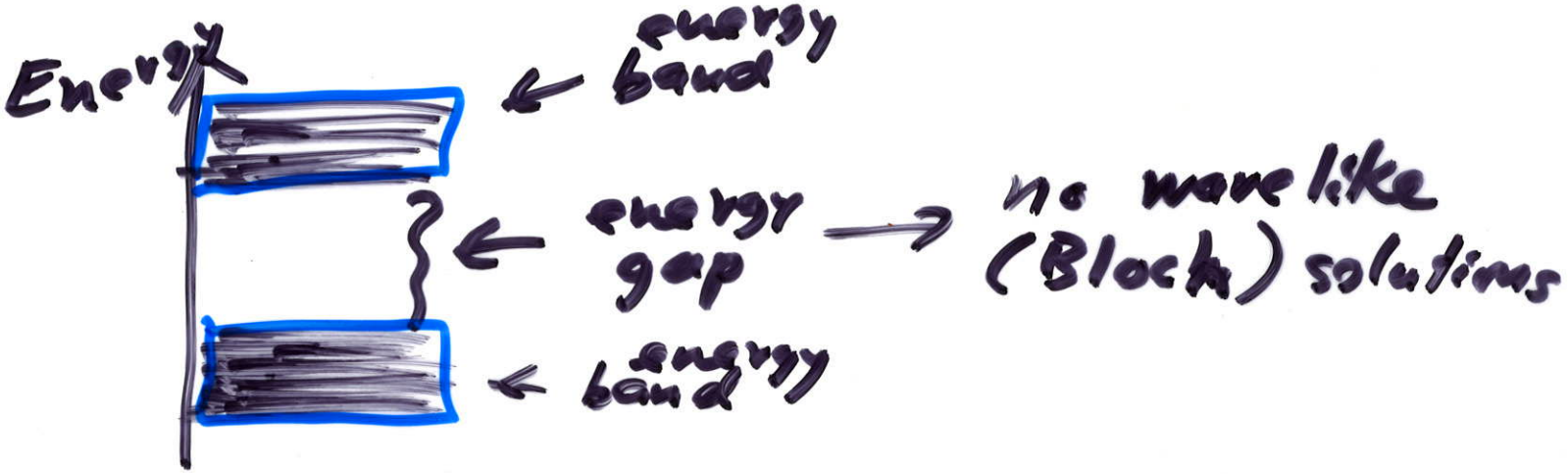
- Quantum Hall Effect
- fractional QHE

Energy Bands

Last chapter- Free electron Fermi gas

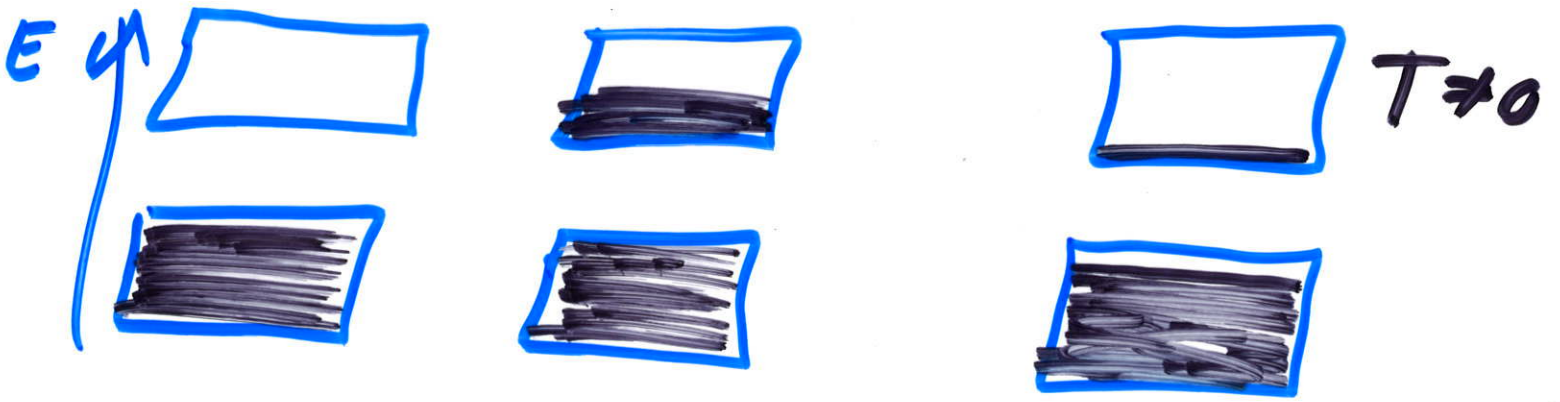
Nearly free electron Fermi gas

Now electrons will interact (weakly) with the crystal lattice.



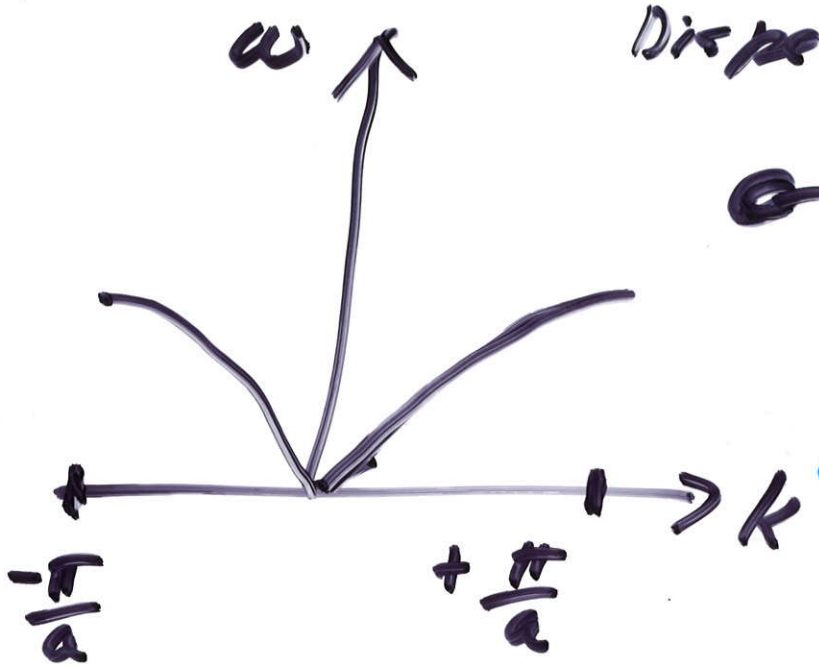
horizontal direction means nothing

Insulator Metal Semiconductor



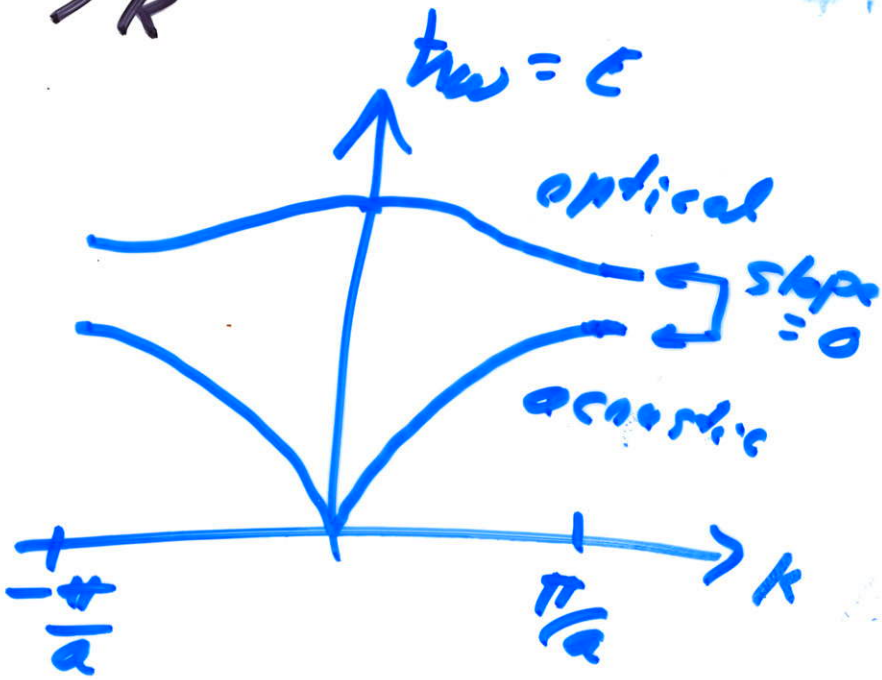
Have seen band gap before **Phonons**

Dispersion Relation

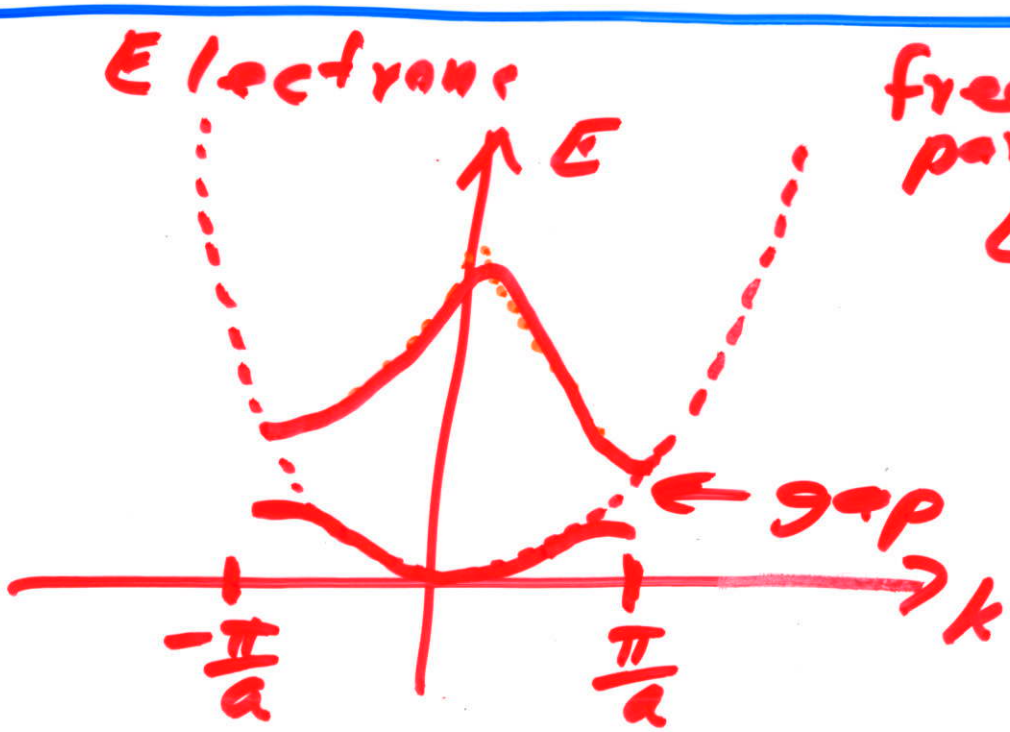


1-atom basis

2-atom basis



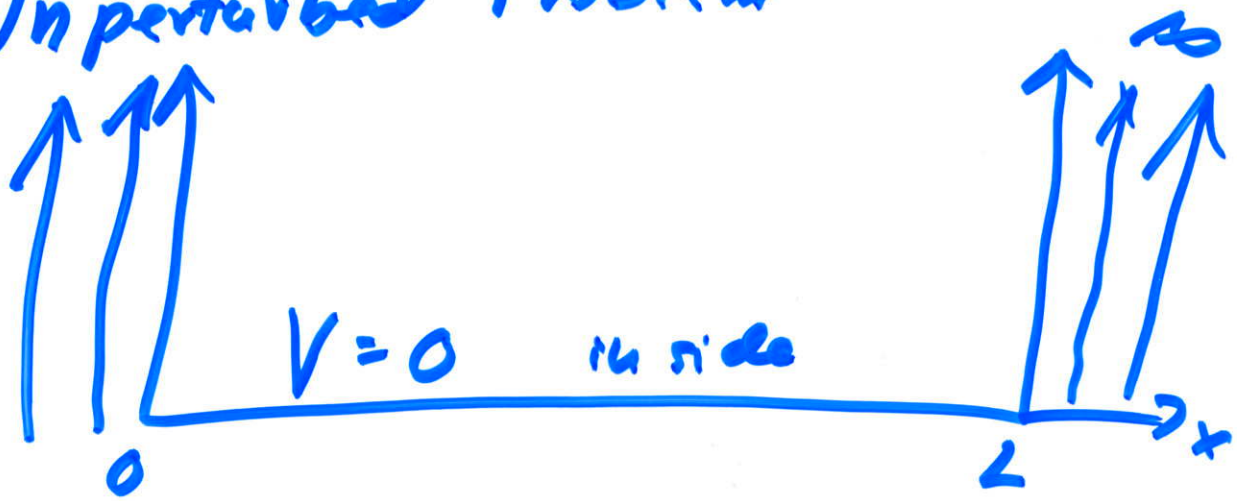
Electrons



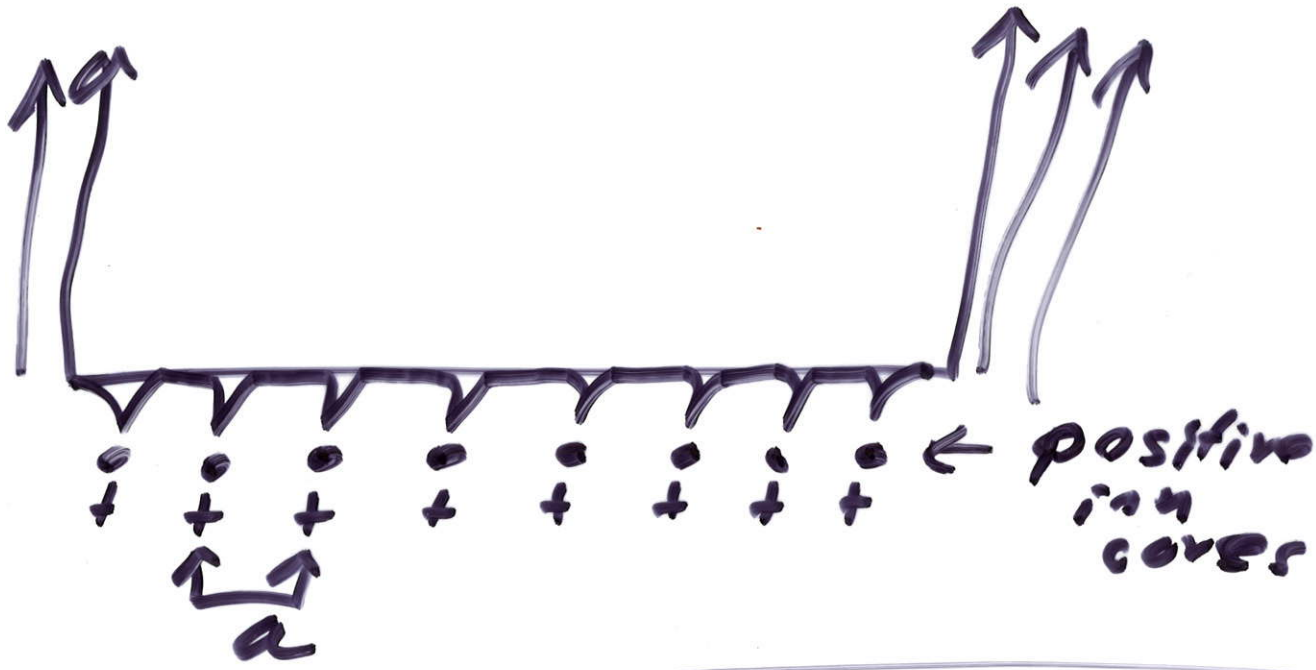
free electrons particle in a box

$$E = \frac{\hbar^2}{2m} k^2$$

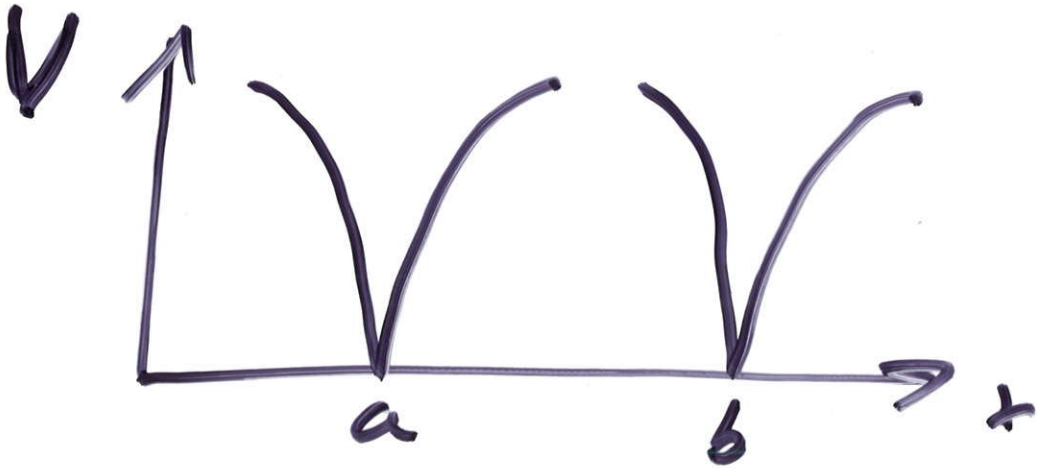
Unperturbed Problem



Perturbation



e.g. Two Hydrogen atoms H_2 molecule



Electron Wave function - overall must be antisymmetric

$$\psi = \psi_{space} \cdot \psi_{spin}$$

-13.6 eV — anti binding / binding

$$\psi_{binding} = \underbrace{\{\psi_a(\vec{r}_1)\psi_b(\vec{r}_2) + \psi_a(\vec{r}_2)\psi_b(\vec{r}_1)\}}_{\text{space - symmetric}} \cdot$$

$$\cdot \frac{1}{\sqrt{2}}(\uparrow_a \downarrow_b - \downarrow_a \uparrow_b) \quad \begin{matrix} S=0 \\ m_s=0 \end{matrix}$$

spin - antisymmetric

singlet

$$\psi = \{\psi_a(\vec{r}_1)\psi_b(\vec{r}_2) - \psi_a(\vec{r}_2)\psi_b(\vec{r}_1)\} \cdot$$

anti-binding space - antisymmetric

$$\cdot \text{(triplet)}$$

$\uparrow_a \uparrow_b$ $\quad S=1, m_s=+1$

$\downarrow_a \downarrow_b$ $\quad S=1, m_s=-1$

$\frac{1}{\sqrt{2}}(\uparrow_a \downarrow_b + \downarrow_a \uparrow_b)$ $\quad S=1, m_s=0$

$N = 2$ Hydrogen atoms

$\Rightarrow$ 2 energy levels

$N \sim N_A \Rightarrow N_A$ energy levels

At the Brillouin zone boundaries

$\rightarrow$ standing waves due to Bragg reflection.

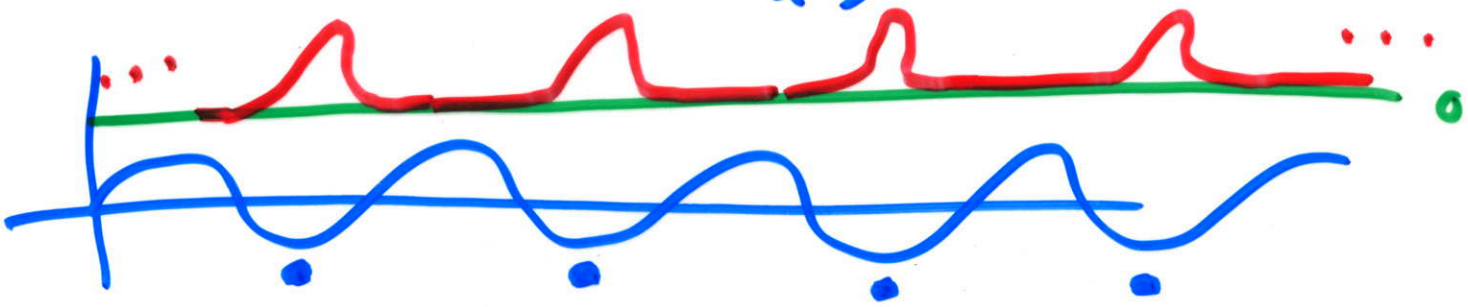
traveling waves $\psi \propto e^{\pm ikx}$

standing waves (at B.Z.)

$\psi_+ \propto e^{ikx} + e^{-ikx} = 2\cos(kx) = 2\cos\left(\frac{\pi}{a}x\right)$

$\psi_- \propto e^{ikx} - e^{-ikx} = 2i\sin(kx) = 2i\sin\left(\frac{\pi}{a}x\right)$

$$U(x) = U\cos\left(\frac{2\pi x}{a}\right)$$



Charge density

$$\rho_+ = |\psi_+|^2 = \psi_+^* \psi_+ \propto \sin^2\left(\frac{\pi x}{a}\right)$$

$$\rho_- = |\psi_-|^2 \propto \sin^2\left(\frac{\pi x}{a}\right)$$

Traveling wave $\psi \propto e^{+ikx}$

$$\rho_0 \propto \psi^*\psi = e^{ikx} \cdot e^{-ikx} = 1$$

energy

$$E_+ = \frac{\int_0^L U(x)\,\rho_+(x)\,dx}{\int_0^L \rho_+(x)\,dx} = +\frac{U}{2}$$

$$E_- = \frac{\int_0^L U(x)\,\rho_-(x)\,dx}{\int_0^L \rho_-(x)\,dx} = -\frac{U}{2}$$