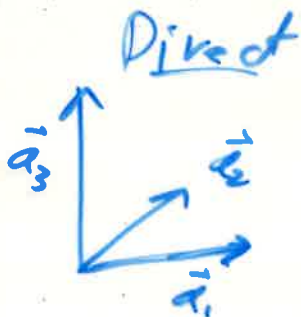


triple product
Volume of p-cell

$$\begin{aligned}
 |\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)| &= |\vec{a}_1 \cdot (\vec{a}_3 \times \vec{a}_2)| \\
 &= |\vec{a}_2 \cdot (\vec{a}_3 \times \vec{a}_1)| = |\vec{a}_2 \cdot (\vec{a}_1 \times \vec{a}_2)| \\
 &= |\vec{a}_3 \cdot (\vec{a}_1 \times \vec{a}_2)| = |\vec{a}_3 \cdot (\vec{a}_2 \times \vec{a}_1)|
 \end{aligned}$$

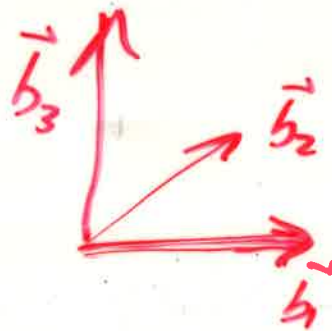
Reciprocal Lattice

ex. SC



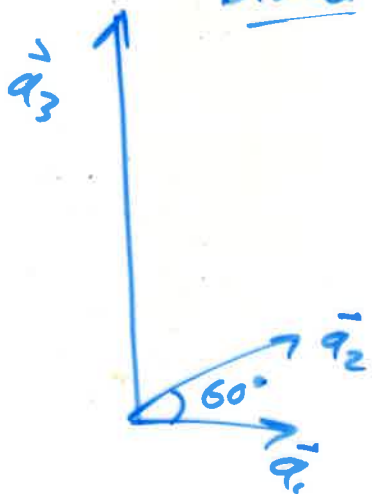
Reciprocal

SC



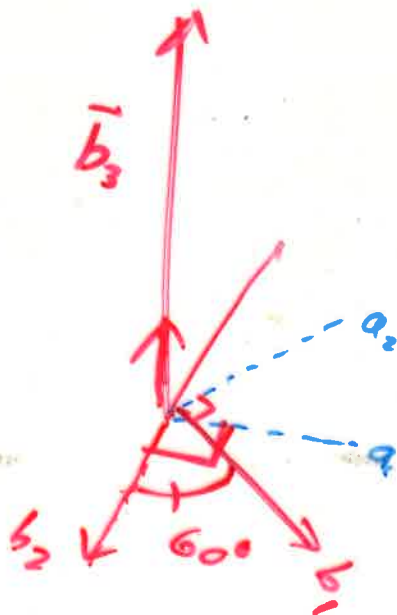
hex

Direct



Reciprocal

hex
(rotated)



Direct
FCC

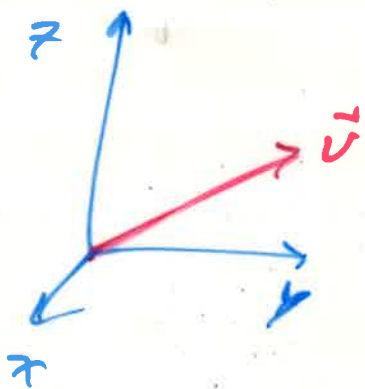
Reciprocal
BCC

BCC

FCC

Reciprocal of Reciprocal is Direct.

Contravariant (upper) indices $\vec{r}, \vec{v}, \vec{a}, \vec{p}, \vec{F}$ Covariant (lower) indices



$$v_x = v_1$$

$$v_i = v^i$$

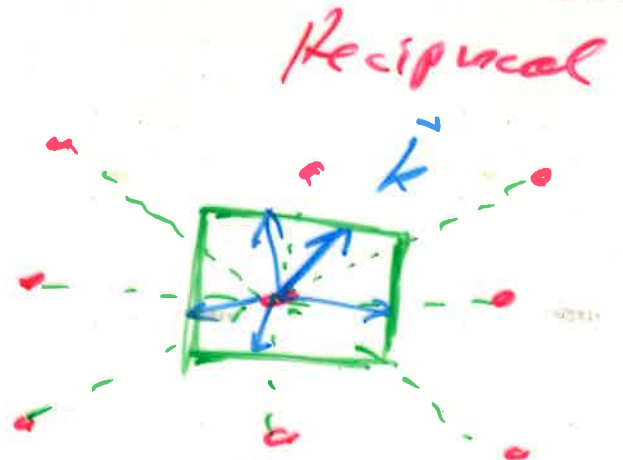


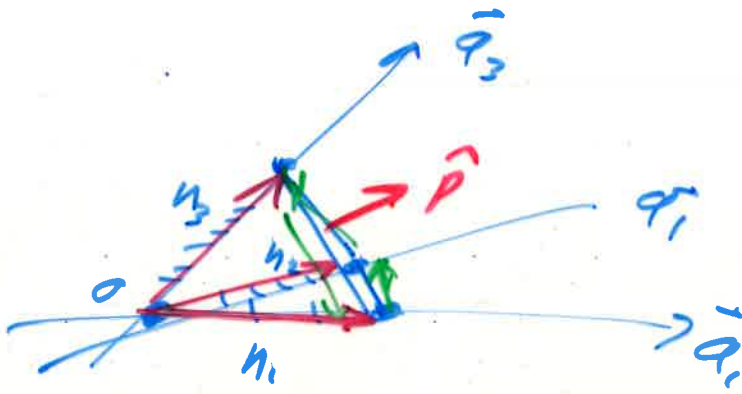
$$v_2 = v^2 = v_3 = v^3$$

Wigner-Seitz cell in Reciprocal Lattice
= Brillouin Zone

First

Direct





2 vectors in plane

$$\vec{v}_1 (n_1 \vec{a}_1 - n_2 \vec{a}_2), \quad \vec{v}_2 (n_2 \vec{a}_2 - n_3 \vec{a}_3)$$

$$\vec{v}_3 (n_3 \vec{a}_3 - n_1 \vec{a}_1)$$

$$\frac{\vec{v}_1 \times \vec{v}_2}{|\vec{v}_1 \times \vec{v}_2|} = \hat{a} \hat{p}$$

$$n_1 \vec{a}_1 \cdot \hat{p} = d$$

$$\frac{1}{n_1} : \frac{1}{n_2} : \frac{1}{n_3} =$$

$$h : k : l$$