Last time

$$I = \int_{y=-\infty}^{+\infty} e^{-y^2}\, dy = \sqrt{\pi}$$

Real(a) > 0

Change variables: $y^2 = a x^2$, $y = \sqrt{a}\, x$, $dy = \sqrt{a}\, dx$

$$y = \pm\infty \Rightarrow x = \pm\infty$$

$$I = \int_{x=-\infty}^{+\infty} e^{-ax^2} \sqrt{a}\, dx = \sqrt{\pi}$$

$$\int_{x=-\infty}^{+\infty} e^{-ax^2} dx = \frac{\sqrt{\pi}}{\sqrt{a}}$$

differentiate both sides w.r.t. a

$$-\int_{x=-\infty}^{+\infty} x^2 e^{-ax^2} dx = \sqrt{\pi}\, \frac{d(a^{-1/2})}{da} = \sqrt{\pi}\left(-\frac{1}{2}\right) a^{-3/2}$$

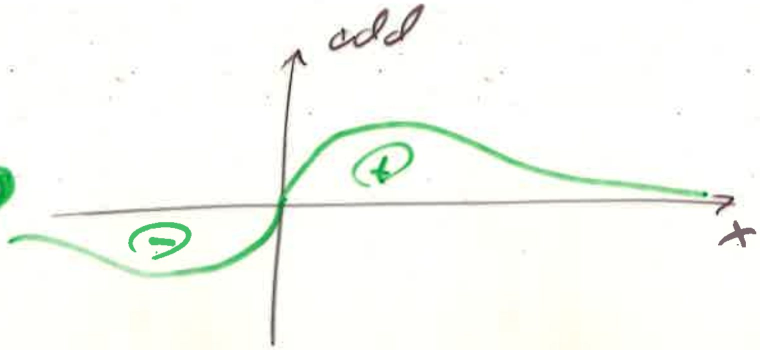
$$\int_{x=-\infty}^{+\infty} x^2 e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$

differentiate w.r.t. a

$$\int_{x=-\infty}^{+\infty} x^4 e^{-ax^2} dx = \frac{3}{4}\sqrt{\frac{\pi}{a^5}}$$

$$\int_{x=-\infty}^{+\infty} x^{2n} e^{-ax^2}\, dx = \text{known}$$

$$\int_{x=-\infty}^{+\infty} x^{2n+1} e^{-ax^2}\, dx = 0$$



$$\langle x \rangle = \frac{\displaystyle\int_{x=-\infty}^{+\infty} e^{\frac{-cx^2}{k_BT}} \left[\cancelto{0}{x} + \frac{gx^4}{k_BT} + \cancelto{0}{\frac{fx^5}{k_BT}} + \cdots \right] dx}{\displaystyle\int_{x=-\infty}^{+\infty} e^{\frac{-cx^2}{k_BT}} \left[1 + \cdots \right] dx}$$

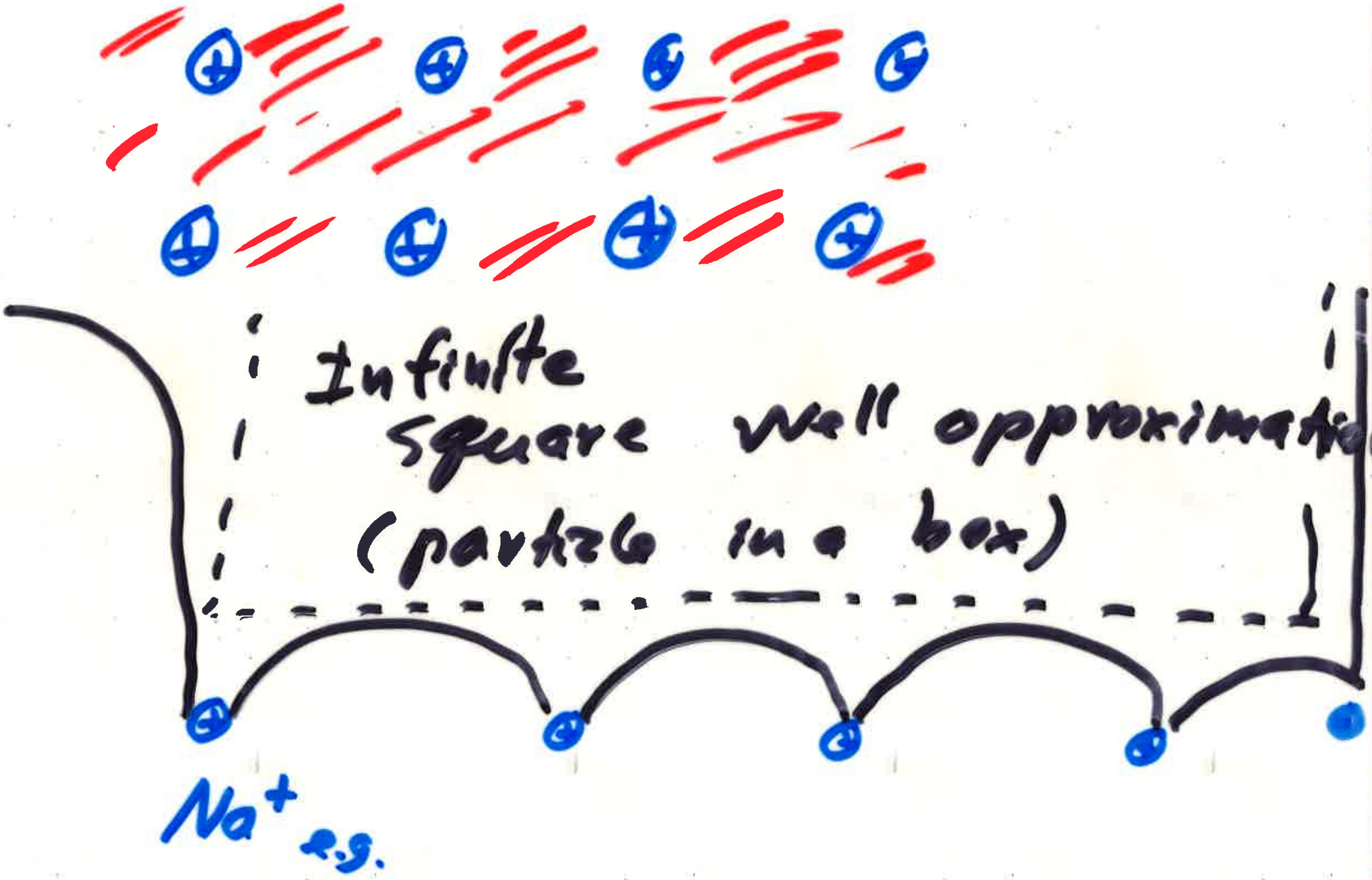
$$\langle x \rangle = \frac{\displaystyle\frac{g}{k_BT} \int_{-\infty}^{+\infty} x^4 e^{-\left(\frac{c}{k_BT}\right)x^2} dx}{\displaystyle\int_{-\infty}^{+\infty} e^{-\left(\frac{c}{k_BT}\right)x^2} dx} = \frac{\displaystyle\frac{g}{k_BT}\,\frac{3}{4}\sqrt{\frac{\pi\, c^{-5}}{(k_BT)^{-5}}}}{\displaystyle\sqrt{\frac{\pi\, k_BT}{c}}}$$

$$\langle x \rangle = \frac{3g}{4c^2} k_BT$$

If we kept the g and f terms in the denominator, this would be

$$\frac{3g}{4c^2} k_BT \left[1 + f(?) \right]$$

and gf is $(\text{small})^2$.



Schroedinger Equation is
non-relativistic, spin-zero

$\hat{x} = x$

$$T + V = H$$

$$\frac{p^2}{2m} + V = H \qquad \hat{p}_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial}{\partial t}\Psi$$

$H\Psi = E\Psi$ for stationary states

Schrödinger (Schroedinger)
Equation

Non-relativistic Spin-zero.

$$\hat{P} = \frac{\hbar}{i}\frac{\partial}{\partial x} \quad , \quad \hat{x} = x \quad , \quad \hat{H} = \hbar i \frac{\partial}{\partial t}$$

$$\frac{\hat{P}^2}{2m}\Psi_{(x,t)} + V(\hat{x})\,\Psi = \hat{H}\,\Psi$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi_{(x,t)} + V(\hat{x})\,\Psi = \hbar i \frac{\partial}{\partial t}\Psi_{(x,t)}$$

Look for "stationary states" = states of definite energy = eigenstates of energy.

$$\hat{H}\,\Psi = E\,\Psi$$

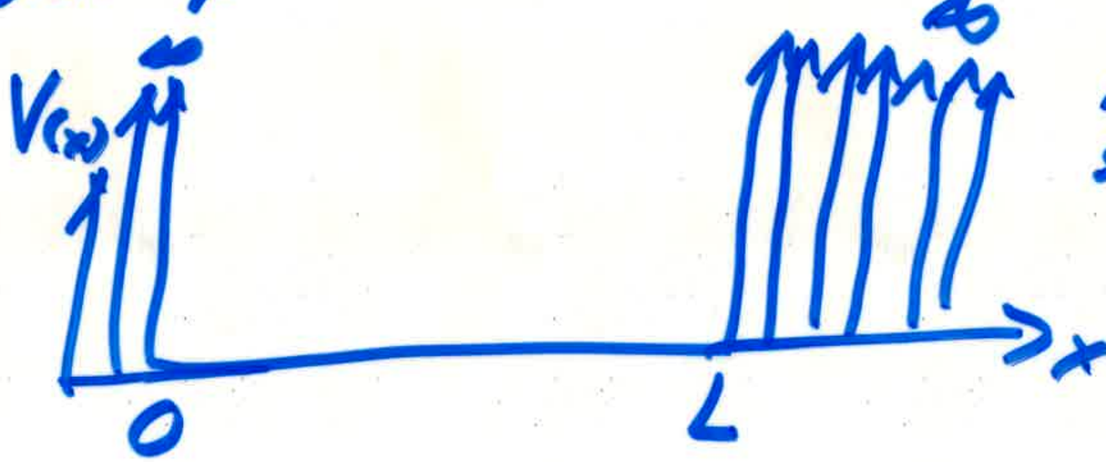
time dependence $e^{\frac{iEt}{\hbar}}$

$$\Psi(x,t) = e^{\frac{iEt}{\hbar}}\,\psi_{(x)}$$

Time-independent Schrödinger Eq.

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi_{(x)} + V_{(x)}\psi_{(x)} = E\,\psi_{(x)}$$

Our problem: particle in a box



infinite square well

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_{(x)} = E\,\psi_{(x)}$$

Try $\psi_{(x)} = A\sin(kx) + B\cos(kx)$

$$\frac{d}{dx}\psi_{(x)} = Ak\cos(kx) - Bk\sin(kx)$$

$$\frac{d^2}{dx^2}\psi_{(x)} = -Ak^2\sin(kx) - Bk^2\cos(kx)$$

$$= -k^2\,\psi_{(x)}$$

$$+\frac{\hbar^2}{2m}k^2\psi_{(x)} = E\,\psi_{(x)} \longrightarrow E = \frac{\hbar^2k^2}{2m}$$

Boundary Conditions $\psi(0) = 0 = \psi(L)$

$$\psi(x) = A\sin(kx) + B\cos(kx)$$

$$\psi(0) = A\cancelto{0}{\sin(0)} + B\cancelto{1}{\cos(0)} = B = 0$$

$$\psi(x) = A\sin(kx)$$

$$\psi(L) = 0 = A\sin(kL)$$

$$A \neq 0 \implies \sin(kL) = 0$$

$$\implies kL = n\pi \qquad n = 1, 2, 3, \ldots$$

Quantum number

$$k_n = \frac{n\pi}{L}$$

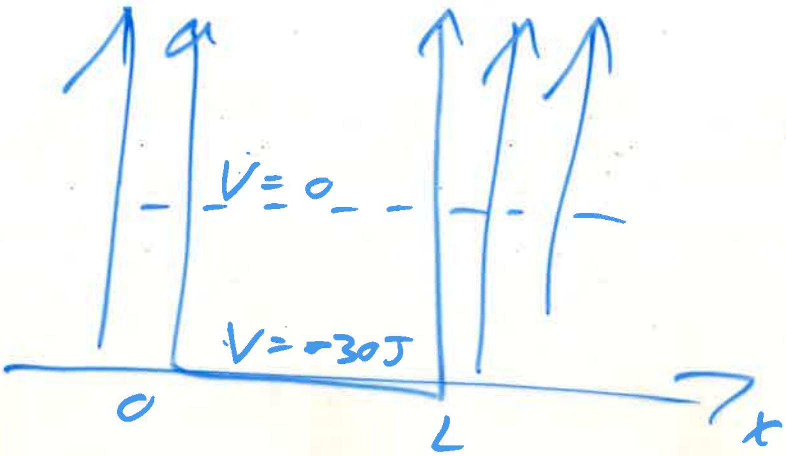
$$\psi(x) = A\sin\left(\frac{n\pi x}{L}\right)$$

$$E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$$

Normalize to get A : $\displaystyle\int_{x=0}^{L} \psi^*(x)\,\psi(x)\,dx = 1$

$$A = \sqrt{\frac{2}{L}}$$

A SQUARE well



Changing the bottom of the square well from $V=0$ to some constant

$$\psi_0 \rightarrow \psi e^{i\theta}$$

introduces a phase into the wave function, but this phase is not physically measurable.

Physical Quantity

$$P(x) = |\psi(x)|^2 = \psi^*(x)\,\psi(x)$$

$$\langle \hat{p} \rangle = \langle \psi_0 | \hat{p} | \psi_0 \rangle$$

AVG momentum