Physics 6351 Professor Scalise
Homework Assignment #2 Spring 2025

1. Along an adiabat, the pressure and volume of an ideal gas are related by $PV^\gamma =$ constant. Show that
$$\frac{dT}{dP} = F(\gamma)\frac{T}{P}$$
and find the function $F(\gamma)$.

2. Helium gas is heated in a process for which the molar heat capacity c_X (neither c_V for constant volume nor c_P for constant pressure) is 2R, where R is the universal gas constant. During the process, the volume of the gas quadruples.

   (a) How does the absolute temperature of the gas change?

   (b) How does the pressure of the gas change?

3. See the attached problem with a PV diagram.

4. See the attached problem on the last page.

Bonus

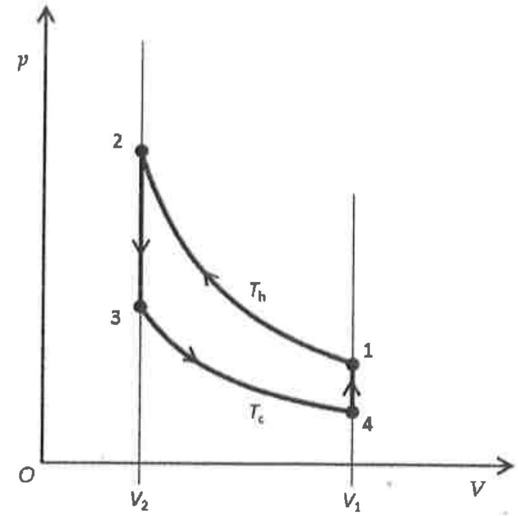
1. (a) Describe and draw all of the quadratic degrees of freedom of a carbon dioxide molecule.

   (b) What is f at high temperature, so that none of the modes are frozen out?

   (c) Treating CO_2 as an ideal gas, what is the molar heat capacity at constant volme in terms of the gas constant R?

   (d) What is the molar heat capacity at constant pressure?

2) The adjacent p-V diagram shows the so-called Stirling cooling cycle (refrigerator). Its working fluid is a monoatomic gas (for instance, helium). Processes 1→2 and 3→4 are isothermal: the fluid is held at constant temperature by thermal baths at temperatures T_h ("hot") and T_c ("cold"), respectively. Processes 2→3 and 4→1 are isochoric: they take place at constant volumes V_2 and V_1, respectively. The heat capacity of the working fluid at constant volume, per mole, is C_V.



Determine the heat absorbed by the fluid in …

a. … the process 1→2
b. … the process 2→3
c. … the process 3→4
d. … the process 4→1

e. Determine the net work done on the fluid per cycle.

f. Is the cycle reversible? Explain.

Problem 1.55. Heat capacities are normally positive, but there is an important class of exceptions: systems of particles held together by gravity, such as stars and star clusters.

(a) Consider a system of just two particles, with identical masses, orbiting in circles about their center of mass. Show that the gravitational potential energy of this system is -2 times the total kinetic energy.

(b) The conclusion of part (a) turns out to be true, at least on average, for *any* system of particles held together by mutual gravitational attraction:

$$\overline{U}_{\text{potential}} = -2\overline{U}_{\text{kinetic}}.$$

Here each $\overline{U}$ refers to the total energy (of that type) for the entire system, averaged over some sufficiently long time period. This result is known as the **virial theorem**. (For a proof, see Carroll and Ostlie (1996), Section 2.4.) Suppose, then, that you add some energy to such a system and then wait for the system to equilibrate. Does the average total kinetic energy increase or decrease? Explain.

(c) A star can be modeled as a gas of particles that interact with each other only gravitationally. According to the equipartition theorem, the average kinetic energy of the particles in such a star should be $\frac{3}{2}kT$, where T is the average temperature. Express the total energy of a star in terms of its average temperature, and calculate the heat capacity. Note the sign.

(d) Use dimensional analysis to argue that a star of mass M and radius R should have a total potential energy of $-GM^2/R$, times some constant of order 1.

(e) Estimate the average temperature of the sun, whose mass is 2×10^{30} kg and whose radius is 7×10^8 m. Assume, for simplicity, that the sun is made entirely of protons and electrons.