1. Read Kardar chapter 2. Did you read all the pages?

2. (a) Use the Gibbs-Duhem relation to write
   i. the enthalpy H
   ii. the Helmholtz free energy F or A

   in terms of T, S, P, V, μ, and N (and nothing else).

   (b) i. Use a Legendre transformation to define a new thermodynamic potential X whose natural variables are (S,V,μ). What is dX?
   ii. Use the Gibbs-Duhem relation to write X in terms of T, S, P, V, μ, and N (and nothing else).

3. A computer bit has two states: 1 and 0. A byte is 8 bits. Your job is to erase a filled terabyte disk.

   (a) How much entropy is created in this irreversible process?

   (b) If this entropy is released to the environment at room temperature, how much heat must have been released to the room?

   (c) Is this amount of energy significant? Compare it to something.

4. Experimental measurements of the heat capacity of aluminum at low temperatures (below about 50 K) can be fit to the formula

$$C_V = aT + bT^3$$

   where C_V is the heat capacity of one mole of Al, $a = 0.00135\ J/K^2$, and $b = 2.48 \times 10^{-}5\ J/K^4$. Find a formula for the entropy per mole of Al in terms of temperature. What is the entropy of 1 gram of Al at 3 K? (Incidentally, the linear piece of C_V is due to the conduction electrons and the cubic piece is due to phonons. Cf. Einstein-Debye specific heat.)

5. A black hole is a region of space where gravity is so strong that nothing can escape. Throwing something into a black hole is therefore an irreversible process. Adding mass to a black hole increases the black hole's entropy. There is no way to tell what kind of matter has gone into making a black hole, therefore the entropy of a black hole must be greater than the entropy of any conceivable type of matter that could have been used to create it. Knowing this, estimate the entropy of a black hole.

   (a) Use dimensional analysis to find the radius of a black hole of mass M in terms of the fundamental constants G and c. (There is, in fact, a 2 in front of the expression which dimensional analysis can not give.)

   (b) What is the entropy of a system (very roughly) in terms of the maximum number of particles N in a system. (Think of an ideal gas or an Einstein solid at high temperature and don't be too concerned with details. You can neglect any logarithms.)

(c) To make a black hole out of the maximum number of particles, you should use particles with the lowest possible energy: long-wavelength photons, but the wavelength can not be any longer than the size of the black hole. By setting the total energy of the photons equal to Mc^2, estimate the maximum number of photons that could be used to make a black hole of mass M.

(d) What is the entropy of a black hole of mass M in terms of its mass? What is the entropy of a black hole of mass M in terms of its area?

(e) What is the entropy of a solar-mass black hole?

(f) What is the entropy of the largest known black hole?

(g) What accounts for most of the entropy of the current Universe?

Bonus

1. Kardar chapter 1 problem 7.