1. Read Kardar sections 3.1, 3.2, and 3.5. Did you read all the pages? Feel free to read the other sections also, but they won't be covered.

2. A cubical die is loaded such that 6 occurs twice as often as 1 (6 and 1 are on opposite sides). Calculate the unbiased probabilities for the six faces of the die by maximizing the entropy $S = -\sum_{i=1}^{6} p_i \ln p_i$ subject to the constraints of unitarity $\sum_{i=1}^{6} p_i = 1$ and the weighting $p_6 = 2p_1$. Use Lagrange multipliers like we did in lecture.

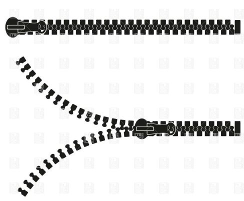
3. Treat the problem of N paramagnetic atoms in an external magnetic field B in contact with a heat bath at absolute temperature T classically, that is, not quantum mechanically. Each atom has magnetic dipole moment μ, but the angle θ that the spin makes with the external magnetic field can take any real value between 0 and π.

   (a) In the absence of an external magnetic field, what is the normalized probability that the polar angle for a single spin in three-dimensional space lies between θ and $\theta + d\theta$? (Remember that the polar angle is not uniformly distrubuted, while the azimuthal angle ϕ is uniformly distributed from zero to 2π.)

   (b) Find the partition function Z.

   (c) Find the average magnetization M as a function of absolute temperature.

   (d) Compare to the answer for a quantized spin-$\frac{1}{2}$ from lecture.



4. Consider a horizontal zipper composed of N links which unzips from left to right. A zipped link has energy 0 and an unzipped link has energy ϵ. A link may only unzip if all of the links to its left are unzipped. (This model is used to study DNA molecules.)

   (a) What is the partition function?

   (b) What is the average number of unzipped links at temperature T?

5. Solve the attached problem.

Bonus

1. A peculiar gas is observed to be governed by two relations:

$$U(V,T) = Vf(T) \qquad \text{and} \qquad \mathrm{P(T)} = \frac{1}{3}\mathrm{f(T)}$$

where U is the energy, P is the pressure, and $f(T)$ is a function of the absolute temperature T. Hints: find dU, use the first law to find dS, look at the mixed partial derivatives noting that dS is an exact differential, obtain a differential equation for $f(T)$.

(a) How does the energy U depend on T?

(b) How does the pressure P depend on T?

(c) How does the entropy S depend on V and T?

Problem 3.34. Polymers, like rubber, are made of very long molecules, usually tangled up in a configuration that has lots of entropy. As a very crude model of a rubber band, consider a chain of N links, each of length ℓ (see Figure 3.17). Imagine that each link has only two possible states, pointing either left or right. The total length L of the rubber band is the net displacement from the beginning of the first link to the end of the last link.

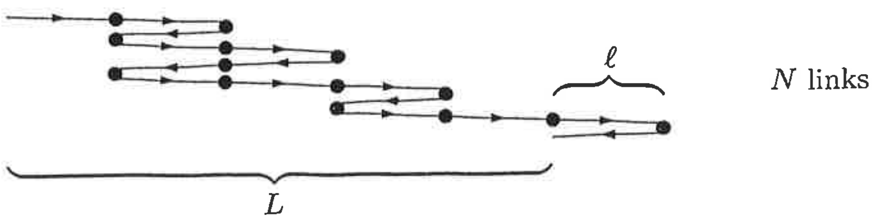


Figure 3.17. A crude model of a rubber band as a chain in which each link can only point left or right.

(a) Find an expression for the entropy of this system in terms of N and N_R, the number of links pointing to the right.

(b) Write down a formula for L in terms of N and N_R.

(c) For a one-dimensional system such as this, the length L is analogous to the volume V of a three-dimensional system. Similarly, the pressure P is replaced by the tension force F. Taking F to be positive when the rubber band is pulling inward, write down and explain the appropriate thermodynamic identity for this system.

(d) Using the thermodynamic identity, you can now express the tension force F in terms of a partial derivative of the entropy. From this expression, compute the tension in terms of L, T, N, and ℓ.

(e) Show that when $L \ll N\ell$, the tension force is directly proportional to L (Hooke's law).

(f) Discuss the dependence of the tension force on temperature. If you increase the temperature of a rubber band, does it tend to expand or contract? Does this behavior make sense?

(g) Suppose that you hold a relaxed rubber band in both hands and suddenly stretch it. Would you expect its temperature to increase or decrease? Explain. Test your prediction with a real rubber band (preferably a fairly heavy one with lots of stretch), using your lips or forehead as a thermometer. (Hint: The entropy you computed in part (a) is not the total entropy of the rubber band. There is additional entropy associated with the vibrational energy of the molecules; this entropy depends on U but is approximately independent of L.)