Physics 7311 Professor Scalise
Homework Assignment #7 Due: 11 March 2005

20. Consider the first full period of the cosine function:$\cos(x)$, $0 < x < 2\pi$.

    (a) Expand this in a Fourier **sine** series and list the first four non-zero Fourier coefficients. (This is not a trick question. You can expand any function outside its given range as either an even or an odd function.)

    (b) Plot the original function and your four-term approximation using a computer for the range $0 < x < 2\pi$.

    (c) Plot the original function and your four-term approximation using a computer for the range $-2\pi < x < 0$. Comment.

21. Mathematica™ Problem: Find the potential $\Phi(x, y)$ which satisfies Laplace's equation subject to the following boundary conditions:

$$\begin{array}{l} y = 0, 0 < x < a : \Phi(x,0) = \Phi_1(x) = V_0\frac{x}{a} \\ x = 0, 0 < y < b : \Phi(0,y) = \Phi_2(y) = V_0\frac{y}{b} \\ y = b, 0 < x < a : \Phi(x,b) = \Phi_3(x) = V_0(1 - \frac{x}{a}) \\ x = a, 0 < y < b : \Phi(a,y) = \Phi_4(y) = V_0(1 - \frac{y}{b}) \end{array}$$

Use the principle of superposition: Solve for the potential $\Phi_1(x, y)$ which satisfies Laplace's equation and has

$$\begin{array}{l} \Phi_1(x) = V_0\frac{x}{a} \\ \Phi_2(y) = 0 = \Phi_3(x) = \Phi_4(y) \quad , \end{array}$$

and similarly for $\Phi_2(x, y)$, $\Phi_3(x, y)$, and $\Phi_4(x, y)$. Then the solution to the original problem is

$$\Phi(x,y) = \Phi_1(x,y) + \Phi_2(x,y) + \Phi_3(x,y) + \Phi_4(x,y) \quad .$$

You can use the symmetry in the problem to obtain one solution from another; for example

$$\Phi_1(x,y;a,b) = \Phi_2(y,x;b,a) \qquad\qquad \Phi_3(x,y;a,b) = \Phi_4(y,x;b,a) \quad .$$

Use Mathematica™ with

$$\begin{array}{l} V_0 = 3 \\ a = 4 \\ b = 5 \end{array}$$

to see if the potential looks like the minimal-energy surface that a soap film would form if stretched on the wire frame below. Try one term in the sum, then five terms, then thirty terms to see the convergence. You can also look at the partial solutions $\Phi_i(x, y)$ for $i = 1, \ldots, 4$ separately for your own amusement.

(over)

The folowing commands might be useful.

NN=30

V0=3

a=4

b=5

Phi[1][x_,y_] = Sum[??? , {n,1,NN}]

Plot3D[Phi[1][x,y], {x,0,a}, {y,0,b}, PlotRange->All]