Electrostatics

MKS

$$\vec{\nabla} \times \vec{E} = 0$$

$$\rho(\vec{r})$$

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho(\vec{r}\,')\, dv'}{|\vec{r} - \vec{r}\,'|}$$

$$\rho(\vec{r}) = -\nabla^2 \Phi(\vec{r})\, \epsilon_0$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho(\vec{r}\,')(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}\, dv'$$

$$\rho(\vec{r}) = \epsilon_0\, \vec{\nabla} \cdot \vec{E}(\vec{r})$$

$$\Phi(\vec{r}) - \Phi(\vec{a}) = -\int_{\vec{a}}^{\vec{r}} \vec{E}(\vec{r}\,') \cdot d\vec{l}\,'$$

$$\Phi(\vec{r}) \qquad\qquad \vec{E}(\vec{r})$$

$$\vec{E}(\vec{r}) = -\vec{\nabla} \Phi(\vec{r})$$

Magnetostatics

MKS

$\vec{J} \Leftarrow \vec\nabla\cdot\vec{J} = 0$

$\vec{B} = \vec\nabla\times\vec{A} \rightarrow \vec\nabla\times\vec{B}$

$\vec\nabla\cdot\vec{B} = 0$

$\vec{J}(\vec{r})$

$\vec{A}_0(\vec{r}) = \frac{\mu_0}{4\pi}\iiint \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}\,dV'$

$\vec{J}(\vec{r}) = \frac{-\nabla^2\vec{A}_0(\vec{r})}{\mu_0}$

Ampere

$\vec{J} = \frac{1}{\mu_0}\vec\nabla\times\vec{B}$

$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\iiint \frac{\vec{J}(\vec{r}')\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,dV'$

$\vec{A}(\vec{r}) = \frac{1}{4\pi}\iiint \frac{\vec{B}(\vec{r}')\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,dV'$

$\vec{A}(\vec{r})$ $\qquad$ $\vec{B}(\vec{r})$

$\vec{B}(\vec{r}) = \vec\nabla\times\vec{A}_0(\vec{r})$

$\vec\nabla\cdot\vec{A}_0(\vec{r}) = 0$

Biot-Savart

38. Since $\vec{\nabla} \cdot \vec{J}(\vec{r}) = 0$, we can write the current density $\vec{J}(\vec{r})$ as the curl of a vector field $\vec{\psi}(\vec{r})$. Use $\vec{J}(\vec{r}) = \vec{\nabla} \times \vec{\psi}(\vec{r})$ to show that

$$\int dV \, [x_\ell J_j(\vec{r}) + x_j J_\ell(\vec{r})] = 0 \ .$$

$$I = \int dV \left\{ x_\ell J_j(\vec{r}) + x_j J_\ell(\vec{r}) \right\}$$

$$= \int dV \left\{ x_\ell \left[\vec{\nabla} \times \vec{\psi}(\vec{r})\right]_j + x_j \left[\vec{\nabla} \times \vec{\psi}(\vec{r})\right]_\ell \right\}$$

$$= \int dV \sum_{i=1}^{3} \sum_{k=1}^{3} \left\{ x_\ell \, \epsilon_{jik} \frac{\partial}{\partial x_i} \psi_k(\vec{r}) + x_j \, \epsilon_{\ell ik} \frac{\partial}{\partial x_i} \psi_k(\vec{r}) \right\}$$

Integrate each term by parts and use the following:

$$\int dV \, x_\ell \frac{\partial}{\partial x_i} \psi_k(\vec{r}) = \int dV \frac{\partial}{\partial x_i}\left[x_\ell \psi_k(\vec{r})\right] - \int dV \, \psi_k(\vec{r}) \frac{\partial x_\ell}{\partial x_i}$$

$$= -\int dV \, \psi_k(\vec{r}) \, \delta_{\ell i}$$

The total derivative term becomes a surface integral which vanishes by the standard arguments.

$$I = -\sum_{i=1}^{3} \sum_{k=1}^{3} \int dV \left\{ \epsilon_{jik} \, \delta_{\ell i} + \epsilon_{\ell ik} \, \delta_{ji} \right\} \psi_k(\vec{r})$$

$$= -\sum_{k=1}^{3} \int dV \left\{ \epsilon_{j\ell k} + \epsilon_{\ell jk} \right\} \psi_k(\vec{r}) = 0$$

since $\epsilon_{j\ell k}$ is totally antisymmetric. ■