Physics 7311 Professor Scalise
Homework Assignment #2 Fall 2022

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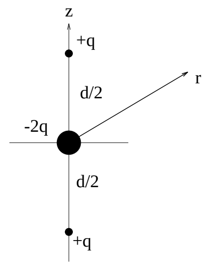
1. Show how to recover the field for the discrete case from the expression for the field in the continuous case using $\rho(\vec{r}) = \sum_i q_i \delta^3(\vec{r} - \vec{r}_i)$.

2. Prove that
$$\frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3} = -\vec{\nabla}\left(\frac{1}{|\vec{r} - \vec{r}\,'|}\right) \quad .$$
This is most easily done by writing $\vec{r} = \sum_i x_i \vec{e_i}$, $\vec{r}\,' = \sum_i x_i' \vec{e_i}$ (with $\vec{e_i} \cdot \vec{e_j} = \delta_{ij}$), and noting that in Cartesian coordinates $\vec{\nabla} = \sum_i \vec{e_i} \frac{\partial}{\partial x_i}$.



3. Consider the distribution of point charges shown in the diagram.
   (a) What is the charge density $\rho(\vec{r})$ for the system?
   (b) What is the potential $\Phi(\vec{r})$?
   (c) Find the leading non-zero term for r≫d.
   (d) What is the power of $(\frac{d}{r})$ in the next term after the one above?
   (e) Take the limits $q \to \infty$ and $d \to 0$ so that the first term is finite but all other terms vanish ("point quadrupole"). What must be the relationship between q and d for this result to occur?

4. Calculate the first three multipole tensors (monopole, dipole, and quadrupole) of a uniformly charged spheroid whose symmetry semi-axis is a and whose transverse semi-axis is b. The charge density ρ is constant within the spheroid and zero without. The equation for the surface of the spheroid is
$$\begin{aligned} x &= b \sin\theta \cos\phi \\ y &= b \sin\theta \sin\phi \\ z &= a \cos\theta \end{aligned} \quad \text{or} \quad \frac{x^2}{b^2} + \frac{y^2}{b^2} + \frac{z^2}{a^2} = 1 \quad .$$