Physics 7311 Professor Scalise
Homework Assignment #5 Fall 2022

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11. Suppose we have an assembly of uniform balls of charge. Then $\rho(\vec{r}) = \sum_i q_i \Delta^3(\vec{r} - \vec{r}_i)$ where

$$\Delta^3(\vec{r} - \vec{r}_i) = \begin{cases} \frac{1}{\frac{4}{3}\pi a^3}, & \vec{r} \text{ in a sphere of radius } a \text{ centered at } \vec{r}_i \\ 0, & \vec{r} \text{ outside the sphere defined above} \end{cases}$$

Thus (assuming that $|\vec{r}_i - \vec{r}_j| > 2a$ for $i \neq j$) the electrostatic potential energy of the system is

$$W = \frac{1}{2}\sum_{i\neq j} q_i q_j \int dV \int dV' \frac{\Delta^3(\vec{r} - \vec{r}_i)\Delta^3(\vec{r}\,' - \vec{r}_j)}{|\vec{r} - \vec{r}\,'|} + \frac{1}{2}\sum_i q_i^2 \int dV \int dV' \frac{\Delta^3(\vec{r} - \vec{r}_i)\Delta^3(\vec{r}\,' - \vec{r}_i)}{|\vec{r} - \vec{r}\,'|} \quad .$$

The last contribution is the sum of **self-energies** of the uniform balls of charge. Calculate the self-energy of the ith ball of charge – i.e. calculate

$$W_i^{\text{self}} = \frac{1}{2} q_i^2 \int dV \int dV' \frac{\Delta^3(\vec{r} - \vec{r}_i)\Delta^3(\vec{r}\,' - \vec{r}_i)}{|\vec{r} - \vec{r}\,'|} \quad .$$

What happens when $a \to 0$? Hint: Shift the origin of coordinates and then put W_i^{self} in the form

$$W_i^{\text{self}} = \frac{1}{2}\int dV \rho_i(\vec{r})\Phi_i^{\text{self}}(\vec{r})$$

with

$$\rho_i(\vec{r}) = q_i \Delta^3(\vec{r})$$

and

$$\Phi_i^{\text{self}}(\vec{r}) = \int dV' \frac{\rho_i(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}$$

and calculate $\Phi_i^{\text{self}}(\vec{r})$ from Gauss' Law.

12. Prove that $C_{ij} = C_{ji}$ where

$$C_{ij} = -\oint_{S_i} \frac{dS}{4\pi} \oint_{S_j} \frac{dS'}{4\pi} \hat{n}_i \cdot \vec{\nabla}_r \;\; \hat{n}_j' \cdot \vec{\nabla}_{r'} G_D(\vec{r}, \vec{r}\,') \quad .$$

13. We often refer to the capacitance C of a pair of conductors. In particular, we put $q_1 = Q$ and $q_2 = -Q$. (equal and oppositely charges conductors). The capacitance is defined as

$$C = \frac{Q}{\Phi_1 - \Phi_2} \quad .$$

(a) Determine C in terms of C_{11}, C_{22}, and C_{12}.

(b) Suppose that conductor #1 is completely enclosed by conductor #2. Prove that $C_{12} = -C_{11}$ in this case. Find the capacitance in this case. Hint: We require $\Phi(\vec{r})$ (the potential at a point $\vec{r}$ due to both conductors) to vanish as $r \to \infty$. With Q on conductor #1 and $-Q$ on conductor #2, what is the electric field outside conductor #2? What does that say about Φ_2?

(c) Consider the case of two concentric spheres of radii R_1 and R_2 with $R_1 < R_2$ and on which there are charges q_1 and q_2, respectively. Do **NOT** assume $q_1 = -q_2$! Find explicit expressions for C_{11}, C_{22}, and C_{12} using elementary physics (e.g. Gauss' Law) together with definitions of the C_{ij} ($q_i = \sum_j C_{ij}\Phi_j$). Comment on your results in light of part (b).

(d) What simplification takes place in part (a) if the two conductors are identical in shape and size?

Bonus:(5 points)

Physics Challenge for Teachers and Students

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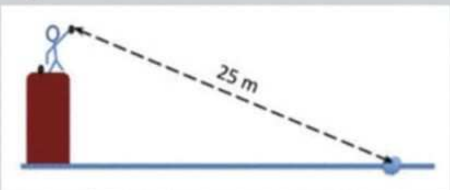
Second to one

A student throws two rocks from the top of a cliff at the same initial speed. The rocks are thrown exactly one second apart, and land simultaneously at the same spot on the ground, 25 meters away from the launch point, as shown in the diagram. Find the flight time of the second rock.

Use $g = 10\ m/s^2$.



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