Physics 7311 | Professor Scalise
Homework Assignment #8 | Fall 2022

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21. In lecture, we claimed that the error

$$E_N(c_1, \ldots, c_N) \equiv \int_a^b d\xi \, |f(\xi) - f_N(\xi)|^2 = \int_a^b d\xi \, [f(\xi) - f_N(\xi)]^*[f(\xi) - f_N(\xi)]$$

made in the approximation

$$f_N(\xi) = \sum_{n=1}^{N} c_n u_n(\xi)$$

to the function $f(\xi)$ was minimized if the expansion coefficients were chosen as

$$c_n = \int_a^b d\xi \, u_n^*(\xi) f(\xi) \quad .$$

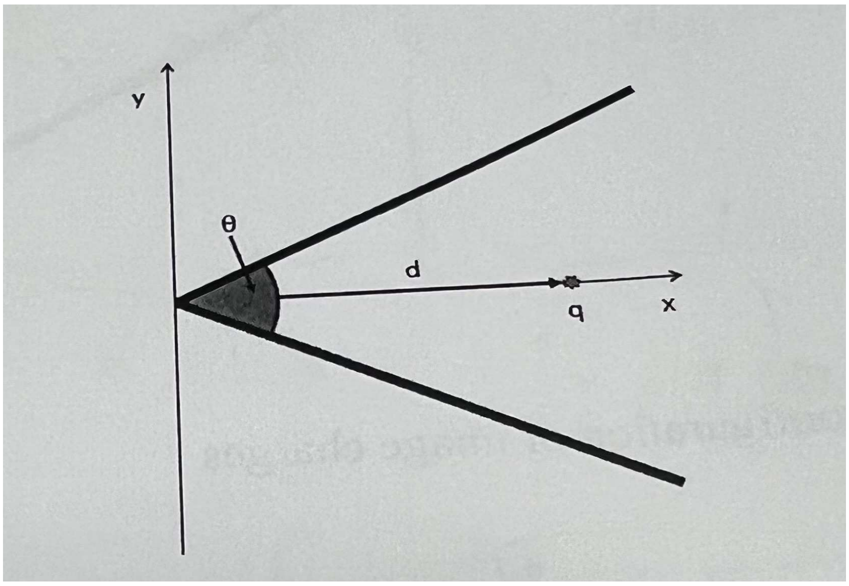
Prove this assertion. Hint: Write $c_n = a_n + ib_n$ where a_n and b_n are real constants, and $\int_a^b d\xi \, u_n^*(\xi) f(\xi) = A_n + iB_n$ where A_n and B_n are real constants. Then require that

$$\frac{\partial E_N}{\partial a_k} = 0 = \frac{\partial E_N}{\partial b_k} \quad .$$

22. Find the potential $\Phi(x, y, z)$ inside a parallelopiped $(0 < x < a, \; 0 < y < b, \; 0 < z < c)$ for which the x-component of the electric field is specified on the face at $x = 0$, and the others faces are grounded.

$$E_x(0, y, z) = f(y, z)$$
$$\Phi(a, y, z) = 0 = \Phi(x, 0, z) = \Phi(x, b, z) = \Phi(x, y, 0) = \Phi(x, y, c)$$

Bonus:(5 points) Due when the homework is due.



Two perfectly conducting infinite and grounded half-planes intersect at an angle $\pi/3$ radians. A charge q is located halfway between the planes at a distance d from the intersection. Find the potential energy of the configuration.