Physics 7311 Professor Scalise
Homework Assignment #7

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1. (a) Consider the first full period of the cosine function:$\cos(x)$, $0 < x < 2\pi$. Expand this in a Fourier **sine** series and list the first four non-zero Fourier coefficients. (This is not a trick question. You can expand any function outside its given range as either an even or an odd function.)

   (b) Plot the original function and your four-term (at least) approximation **using a computer** for the range $0 < x < 2\pi$.

   (c) Expand $\cos(x)$, $0 < x < 2\pi$, in a Fourier cosine series. Find all the coefficients.

2. Find the potential $\Phi(x, y)$ which satisfies Laplace's equation subject to the following boundary conditions:

$$\begin{aligned} y = 0, 0 < x < a &: \Phi(x,0) = \Phi_1(x) = V_0 \tfrac{x}{a} \\ x = 0, 0 < y < b &: \Phi(0,y) = \Phi_2(y) = V_0 \tfrac{y}{b} \\ y = b, 0 < x < a &: \Phi(x,b) = \Phi_3(x) = V_0(1 - \tfrac{x}{a}) \\ x = a, 0 < y < b &: \Phi(a,y) = \Phi_4(y) = V_0(1 - \tfrac{y}{b}) \end{aligned}$$

Use the principle of superposition: Solve for the potential $\Phi_1(x, y)$ which satisfies Laplace's equation and has

$$\begin{aligned} &\Phi_1(x) = V_0 \tfrac{x}{a} \\ &\Phi_2(y) = 0 = \Phi_3(x) = \Phi_4(y) \ \ , \end{aligned}$$

and similarly for $\Phi_2(x, y)$, $\Phi_3(x, y)$, and $\Phi_4(x, y)$. Then the solution to the original problem is

$$\Phi(x, y) = \Phi_1(x, y) + \Phi_2(x, y) + \Phi_3(x, y) + \Phi_4(x, y) \ \ .$$

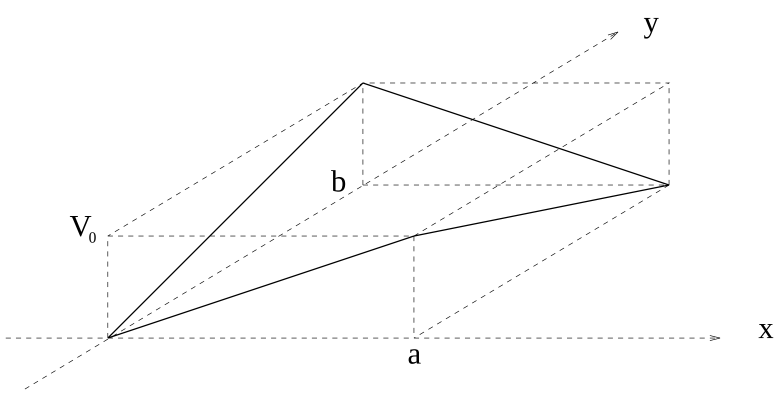
You can use the symmetry in the problem to obtain one solution from another; for example

$$\Phi_1(x, y; a, b) = \Phi_2(y, x; b, a) \qquad\qquad \Phi_3(x, y; a, b) = \Phi_4(y, x; b, a) \ \ .$$

Use computer software with

$$\begin{aligned} V_0 &= 3 \\ a &= 4 \\ b &= 5 \end{aligned}$$

to see if the potential looks like the minimal-energy surface that a soap film would form if stretched on the wire frame below. Try one term in the sum, then five terms, then thirty terms to see the convergence.



Bonus

1. Prove Newton's Theorem: Find the electric field (Newton of course did this for the gravitational field) a distance z from the center of a spherical shell of radius R which carries charge q uniformly spread over its surface. Do not use Gauss' law (because that assumes Newton's Theorem which you are trying to prove). Perform the integrals explicitly and treat the cases $z < R$ (inside) and $z < R$ (outside) separately.

   Hint: use the law of cosines to write $|\vec{r} - \vec{r'}|$ in terms of R and θ. (The problem has azimuthal symmetry and therefore no dependence on ϕ.) Be sure to take the *positive* square roots when appropriate. For example, $\sqrt{R^2 + z^2 - 2Rz} = (R - z)$ if $R > z$, but it's $(z - R)$ if $R < z$.