Physics 7311 Professor Scalise
Homework Assignment #9

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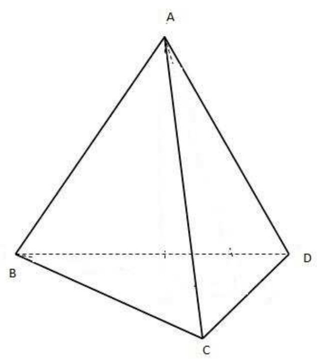
1. Consider a ring of radius $a$ in the xy-plane with center at the origin whose linear charge density is $\lambda(\phi) = \frac{Q}{a}\cos\phi$. Find the first three **spherical** multipole tensors $Q_{\ell m}$ ($\ell = 0, 1, 2$ and $-\ell \leq m \leq \ell$).

2. For each of the following, state whether or not the expression satisfies the Cauchy-Riemann equations. If it does, rewrite the expression as a function of $z$ (and not $z^*$).
   (a) $e^x[\cos(y) + i\sin(y)]$
   (b) $e^{x^2-y^2}[\cos(2xy) + i\sin(2xy)]$
   (c) $\cos(x)\cosh(y) - i\sin(x)\sinh(y)$
   (d) $x^2 + y^2 + 2ixy$

3. Show that $z(f) = \frac{a}{2\pi}(1 + f + e^f)$ is analytic by satisfying the Cauchy-Riemann equations. Show that the inverse map $f(z)$ is analytic by satisfying the Cauchy-Riemann equations. You will not be able to isolate $f$, so you need to be clever.

## Bonus:




If each vertex of a tetrahedron is connected by a resistor $R$ (6 in total), what is the equivalent resistance from one vertex to another?

If each vertex of a cube is connected by a resistor $R$ (12 in total), what is the equivalent resistance from one vertex to an adjacent vertex (connected by a cube edge)?