Physics 7311 Professor Scalise
Homework Assignment #12

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1. An infinitely long cylindrical shell of radius a carries current I along the axis of the cylinder ($\hat{z}$) . In what follows, you may need to use generalized functions (distributions) and the curl and divergence in cylindrical coordinates:

$$\vec{\nabla} \times \vec{f}(s, \phi, z) = \left(\frac{1}{s} \frac{\partial f_z}{\partial \phi} - \frac{\partial f_\phi}{\partial z} \right) \hat{s} + \left(\frac{\partial f_s}{\partial z} - \frac{\partial f_z}{\partial s} \right) \hat{\phi} + \frac{1}{s} \left(\frac{\partial (s f_\phi)}{\partial s} - \frac{\partial f_s}{\partial \phi} \right) \hat{z}$$

$$\vec{\nabla} \cdot \vec{f}(s, \phi, z) = \frac{1}{s} \frac{\partial (s f_s)}{\partial s} + \frac{1}{s} \frac{\partial f_\phi}{\partial \phi} + \frac{\partial f_z}{\partial z}$$

   (a) What is the current density vector field $\vec{J}(\vec{r})$ everywhere?

   (b) What is the magnetic vector field $\vec{B}(\vec{r})$ everywhere?

   (c) What is the vector potential field $\vec{A}(\vec{r})$ in Coulomb gauge everywhere? Make sure you check $s < a$, $s > a$, and $s = a$.

2. An infinitely long cylindrical shell solenoid of radius a has magnetic field $\vec{B}$ constant inside parallel to the symmetry axis of the solenoid and zero outside the solenoid. In what follows, you may need to use generalized functions (distributions) and the curl and divergence in cylindrical coordinates. Remember to answer in terms of the givens, for example B, not I or J.

   (a) What is the current density vector field $\vec{J}(\vec{r})$ everywhere?

   (b) What is the vector potential field $\vec{A}(\vec{r})$ in Coulomb gauge everywhere? Make sure you check $s < a$, $s > a$, and $s = a$.

Bonus:

The upper half space $z > 0$ is filled with dielectric ϵ_1 while the lower half space $z < 0$ is filled with dielectric ϵ_2. A true point charge q sits at the point $(x, y, z) = (0, 0, d)$.

1. Find the bound surface polarization charge density $\sigma(x, y)$ on the interface between the dielectrics. (Cylindrical polar coordinates might be more convenient.)

2. Integrate the bound surface polarization charge density found above to find the total bound charge on the interface.