In problem #5 you were asked to calculate the Poynting vector in the case of total internal reflection. In the present context a similar calculation shows that

$$\langle \vec{S} \rangle = \frac{c}{8\pi} \, Re \, (\vec{k} \, \vec{E}_0 \cdot \vec{E}_0^{\,*}) \, e^{-2(Im \, \vec{k}) \cdot \vec{r}}$$

For $\omega > \omega_p$ we have $k = \sqrt{\omega^2 - \omega_p^2} \,/c$ and $\vec{k}$ is a real wave vector. Then $\langle \vec{S} \rangle = \frac{c}{8\pi} \vec{k} \, \vec{E}_0^{*} \cdot \vec{E}_0$. For $\omega < \omega_p$ we have $k = i \sqrt{\omega_p^2 - \omega^2} \,/c$ and $\vec{k}$ is an imaginary wave vector. Hence $\langle \vec{S} \rangle = \vec{0}$ and we get no wave propagation in this case.

A non-propagating non-absorbing wave is called an evanescent wave. The significance of the threshold requirement $\omega > \omega_p$ for wave propagation is "screening" of the electric field. For example, for $\omega = 0$ in a conductor (and in the absence of an emf) there can be no electric field. The above result is an extension to wave propagation in a conducting medium. For $\omega < \omega_p$ the charges (electrons) respond quickly to the electric field and screen it out. For $\omega > \omega_p$ the "plasma" cannot "follow" the electric field and hence cannot screen it out.

Furthermore, since $\vec{k} \cdot \vec{E}_0 = 0$ for this wave mode, there is no charge density — the electrons move back and forth in a shearing manner perpendicular to the direction of the wave propagation.

Finally, plasma oscillations are observed in laboratory plasmas. Just like the electromagnetic field can be <u>quantized</u> and the state of the quantized field can be described by the presence of photons, So these plasma oscillations can also be quantized. For negligible damping we have

transverse plasmons: $\hbar\omega = \sqrt{(\hbar k)^2 c^2 + (\hbar\omega_p)^2}$

plasmon energy $= \hbar\omega$, plasmon momentum $= \hbar k$ (note: transverse plasmons look like a relativistic particle with mass $\hbar\omega_p / c^2$).

longitudinal plasmons: $\hbar\omega = \hbar\omega_p$ = plasmon energy

(note: longitudinal plasmons look like a particle with infinite mass — recall they have no group velocity)

Plasmons can be detected in metals (an "electron gas") — typically $\hbar\omega_p \sim 5-10$ ev. Since $k_B T \ll \hbar\omega_p$, the plasmons are not thermally excited but can be excited by an electron beam. In evolved stars both $\hbar\omega_p$ and $k_B T$ are enormous compared to a metal. The thermally excited plasmons (both longitudinal and transverse) can decay into neutrino-antineutrino pair (through a "virtual" electron-positron pair). Since these neutrinos escape from the star, neutrino emission can be an important way in which evolved stars cool their interiors (there are several other ways in which neutrinos are generated for the cooling process and they are usually more important than the plasmon process).