Now examine two cases:

$E_{02} = 0$ (incident wave linearly polarized perpendicular to the plane of incidence)

the $\boxed{\text{equations}}$ on the previous page $\Rightarrow E_{02}' = 0$ and $E_{02}'' = 0$

The reflected and refracted waves are also polarized perpendicular to the plane of incidence.

The equations involving the 1 components

$$E_{01} + E_{01}'' - E_{01}' = 0$$

$$\frac{n}{\mu} \cos(\theta_i)(E_{01} - E_{01}'') - \frac{n'}{\mu'} \cos(\theta_b) E_{01}' = 0$$

can be solved for the ratios: (Use Snell's Law to eliminate θ_b)

$$\frac{E_{01}'}{E_{01}} = \frac{2n\cos(\theta_i)}{n\cos(\theta_i) + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2(\theta_i)}}$$

$$\frac{E_{01}''}{E_{01}} = \frac{n\cos(\theta_i) - \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2(\theta_i)}}{n\cos(\theta_i) + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2(\theta_i)}}$$

Second case: $E_{01} = 0$ (incident wave linearly polarized in the plane of incidence).

This time $E_{01}' = 0$ and $E_{01}'' = 0$. That is, the reflected and refracted waves are also polarized in the plane of incidence

Two of the three equations involving the z components can be solved for the ratios (remember they are not all linearly independent):

$$\left(E_{0z} - E_{0z}''\right)\cos(\theta_i) - E_{0z}'\cos(\theta_t) = 0$$

$$\frac{n}{\mu}\left(E_{0z} + E_{0z}''\right) - \frac{n'}{\mu'}E_{0z}' = 0$$

$$\frac{E_{0z}'}{E_{0z}} = \frac{2nn'\cos(\theta_i)}{\frac{\mu}{\mu'}n'^2\cos(\theta_i) + n\sqrt{n'^2 - n^2\sin^2(\theta_i)}}$$

$$\frac{E_{0z}''}{E_{0z}} = \frac{\frac{\mu}{\mu'}n'^2\cos(\theta_i) - n\sqrt{n'^2 - n^2\sin^2(\theta_i)}}{\frac{\mu}{\mu'}n'^2\cos(\theta_i) + n\sqrt{n'^2 - n^2\sin^2(\theta_i)}}$$

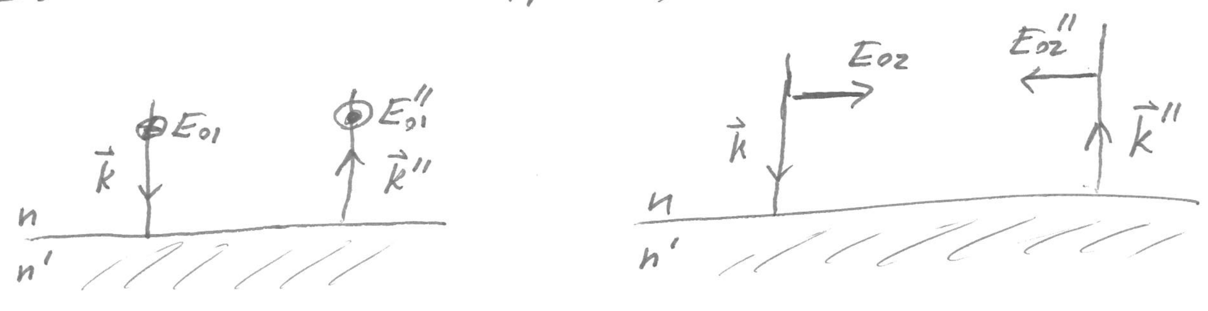
Look at some special cases: Usually, we may set $\mu = \mu'$. (Recall that for static fields in most materials $\mu \approx \mu_0$ except in ferromagnets.)

If we also have <u>normal incidence</u> $\theta_i = 0$, then

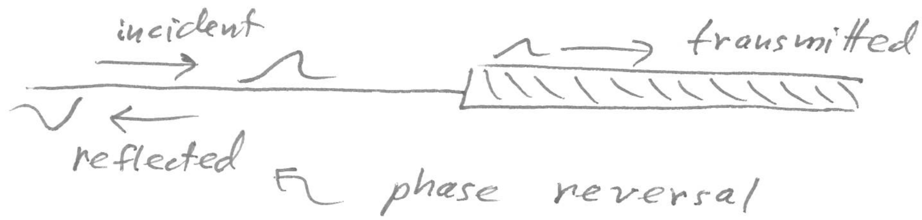
$$\frac{E_{01}'}{E_{01}} = \frac{2n}{n+n'} \qquad \frac{E_{01}''}{E_{01}} = \frac{n-n'}{n+n'}$$

$$\frac{E_{0z}'}{E_{0z}} = \frac{2n}{n+n'} \qquad \frac{E_{0z}''}{E_{0z}} = \frac{n'-n}{n+n'}$$

In both polarization directions for $n' > n$, there is a phase reversal upon reflection. That is, $\vec{E}_0$ and $\vec{E}_0''$ are oppositely directed.



This is similar to what happens when a wave on a rope made of a light piece and a heavy piece travels from the light piece to the heavy piece



Polarization by Reflection $(\mu \approx \mu')$

Q: Is it possible to have the reflected wave amplitude vanish?

For $\vec{E}_1$ polarization, $E_{01}'' = 0 \Rightarrow n\cos(\theta_i) = \sqrt{n'^2 - n^2\sin^2(\theta_i)}$

or $\quad n^2\cos^2(\theta_i) = n'^2 - n^2\sin^2(\theta_i)$

$$n^2[\cos^2(\theta_i) + \sin^2(\theta_i)] = n'^2 \Rightarrow n^2 = n'^2 \Rightarrow n = n'$$

Well, if $n = n'$ there certainly is no reflected wave, but there is also no interface!

For $\vec{E}_2$ polarization, $E''_{0z} = 0 \Rightarrow n'^2 \cos(\theta_i) = n\sqrt{n'^2 - n^2 \sin^2(\theta_i)}$

or $\quad n'^4 \cos^2(\theta_i) = n^2\left[n'^2 - n^2 \sin^2(\theta_i)\right]$

$$n'^4\left[1 - \sin^2(\theta_i)\right] = n^2 n'^2 - n^4 \sin^2(\theta_i)$$

$$n'^2(n'^2 - n^2) = (n'^4 - n^4)\sin^2(\theta_i) = (n'^2 + n^2)(n'^2 - n^2)\sin^2(\theta_i)$$

$$n'^2 = (n'^2 + n^2)\sin^2(\theta_i)$$

$$\Rightarrow \quad \sin(\theta_i) = \frac{n'}{\sqrt{n^2 + n'^2}} \qquad \cos(\theta_i) = \frac{n}{\sqrt{n^2 + n'^2}}$$

$$\tan(\theta_i) = \frac{\sin(\theta_i)}{\cos(\theta_i)} = \frac{n'}{n} \quad \text{or} \quad \boxed{\theta_i = \arctan\left(\frac{n'}{n}\right)}$$

this is Brewster's angle

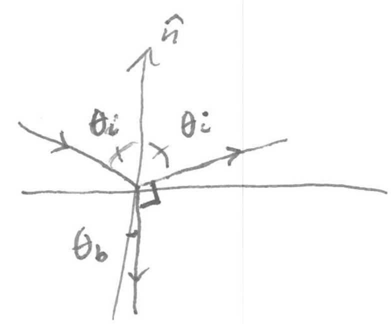
Brewster's angle is the angle of incidence at which an incident wave polarized in the plane of incidence is totally transmitted — there is no reflection.

Thus an unpolarized beam of radiation will become polarized perpendicular to the plane of incidence upon reflection when $\theta_i = \theta_{Brewster}$.

From Snell's Law: $n \sin(\theta_i) = n' \sin(\theta_b)$, it follows that

$$\underset{\text{BREWSTER}}{\sin(\theta_b)} = \underset{\text{Brewster}}{\cos(\theta_i)}$$

or $\quad \theta_i + \theta_b = \frac{\pi}{2}$ at Brewster's angle



Total Internal Reflection

Consider the case $n > n'$. (The incident medium is more optically dense than the refracted medium.)

Snell's Law is $\frac{n}{n'} \sin(\theta_i) = \sin(\theta_b)$

There is a critical angle $\theta_{i_{crit}}$ for which $\theta_b = \frac{\pi}{2}$

$$\sin(\theta_{i_{crit}}) = \frac{n'}{n} < 1 \implies \theta_{i_{crit}} = \arcsin\left(\frac{n'}{n}\right)$$

However, for $\theta_i > \theta_{i_{crit}}$, we have $\sin(\theta_b) > 1$. In this case, the refracted angle becomes <u>complex</u>.

$$\cos(\theta_b) = \sqrt{1 - \sin^2(\theta_b)} = \sqrt{1 - \frac{n^2}{n'^2} \sin^2(\theta_i)}$$

$$= i \sqrt{\frac{n^2}{n'^2} \sin^2(\theta_i) - 1}$$

There are 2 roots, one positive, one negative. We will choose the positive root for exponential <u>decay</u>.

Thus $\cos(\theta_b)$ is <u>purely imaginary</u>.

The plane wave factor $e^{i \vec{k}' \cdot \vec{r}}$ for the refracted wave can be written as

$$e^{i \vec{k}' \cdot \vec{r}} = e^{i \vec{k}' \cdot \hat{n} (\hat{n} \cdot \vec{r})} e^{i \vec{k}'_{\parallel} \cdot \vec{r}_{\parallel}}$$

where $\vec{k}' \cdot \hat{n} = k' \cos(\theta_b) = i k' \sqrt{\frac{n^2}{n'^2} \sin^2(\theta_i) - 1}$

and $\vec{k}'_{\parallel} \equiv \vec{k}' \times \hat{n} \qquad \vec{r}_{\parallel} \equiv \vec{r} \times \hat{n}$

$e^{i\vec{k}_{\parallel}'\cdot\vec{r}_{\parallel}}$ is still a phase factor, but

$$e^{i\vec{k}'\cdot\hat{n}(\hat{n}\cdot\vec{r})} = e^{-k'(\hat{n}\cdot\vec{r})\sqrt{\frac{n^2}{n'^2}\sin^2(\theta_i)-1}}$$

is a <u>decaying exponential</u>. Thus the electric and magnetic fields decay exponentially in the refracted medium.

Homework: The wave vector in the refracted medium for $\theta_i > \theta_{i\,crit}$ is

$$\vec{k}' \equiv \vec{k}_{\parallel}' + i\mathcal{K}\hat{n} \qquad \text{where } \vec{k}_{\parallel}'\cdot\hat{n} = 0$$

and $$\mathcal{K} = \underbrace{k}_{k,\text{ not } k'}\sqrt{\sin^2(\theta_i) - \sin^2(\theta_{i\,crit})}$$

$$\sin(\theta_{i\,crit}) = \frac{n'}{n}$$

$$k = \frac{\omega}{v} \qquad k' = \frac{\omega}{v'} \qquad k' = \frac{v}{v'}k = \frac{n'}{n}k$$

Show that the Poynting vector in the refracted medium averaged over one period is

$$\langle \vec{\mathcal{S}}' \rangle = \frac{1}{2\mu\omega}\left[\vec{k}_{\parallel}'(\vec{E}_0'^{*}\cdot\vec{E}_0') + 2\mathrm{Re}(i\vec{E}_0'^{*}\,\hat{n}\cdot\vec{E}_0')\right]e^{-2\mathcal{K}\hat{n}\cdot\vec{r}}$$

<u>Caution!</u> Since $\vec{k}'$ is complex, you must treat it carefully.

It follows from this result that $\langle \hat{n}\cdot\vec{\mathcal{S}}' \rangle = 0$ since $\vec{E}_0'^{*}\cdot\vec{E}_0'$ is real and $\hat{n}\cdot\vec{k}' = i\mathcal{K}$ is imaginary. Thus no energy is transmitted into the refracted medium (on the average). There is a component parallel to the interface, but it decays exponentially as you look deeper into the refracted medium with penetration depth

$$d = \frac{c}{2\omega} \Big/ \sqrt{n^2 \sin^2(\theta_i) - n'^2}$$

Note that $d \to \infty$ as $\theta_i \to \theta_{i_{crit}}$

The reflected waves for $\vec{\epsilon}_1$ and $\vec{\epsilon}_2$ polarizations become, for $\theta_i > \theta_{i_{crit}}$

$$\frac{E_{01}''}{E_{01}} = \frac{n\cos(\theta_i) - i\frac{\mu}{\mu'}\sqrt{n^2\sin^2(\theta_i) - n'^2}}{n\cos(\theta_i) + i\frac{\mu}{\mu'}\sqrt{n^2\sin^2(\theta_i) - n'^2}}$$

$$\frac{E_{02}''}{E_{02}} = \frac{\frac{\mu}{\mu'} n'^2 \cos(\theta_i) - i n\sqrt{n^2\sin^2(\theta_i) - n'^2}}{\frac{\mu}{\mu'} n'^2 \cos(\theta_i) + i n\sqrt{n^2\sin^2(\theta_i) - n'^2}}$$

So $E_{01}'' = e^{i\psi_1} E_{01}$ and $E_{02}'' = e^{i\psi_2} E_{02}$