Physics 7312 Professor Scalise
Homework Assignment #3 Spring 2023

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1. Since $\vec{\nabla} \cdot \vec{J}(\vec{r}) = 0$, we can write the current density $\vec{J}(\vec{r})$ as the curl of a vector field $\vec{\psi}(\vec{r})$. Use $\vec{J}(\vec{r}) = \vec{\nabla} \times \vec{\psi}(\vec{r})$ to show that the symmetric piece from lecture vanishes:

$$\int dV \; [x_\ell J_j(\vec{r}) + x_j J_\ell(\vec{r})] = 0 \ \ .$$

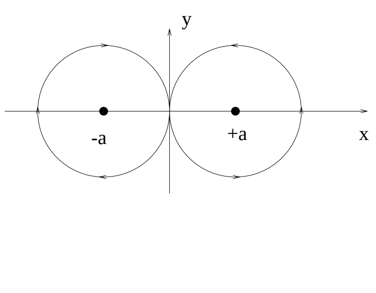
2. The magnetic scalar potential due to a point magnetic dipole at the origin is

$$\Phi_m(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \cdot \vec{r}}{r^3}$$

   where $\vec{m}$ is the magnetic dipole moment.

   Find the Cartesian magnetic dipole moment vector and the reducible and irreducible Cartesian magnetic quadrupole moment tensors of the system consisting of two point magnetic dipoles: $\vec{m}$ at $\vec{a}$ and $-\vec{m}$ at $-\vec{a}$. The directions of $\vec{m}$ and $\vec{a}$ and independent. (Your answer should only contain components of m and a; that is, do not impose a coordinate system of your own choosing on the problem. For example, do not choose the z-axis along $\vec{m}$.)

3. What is the Cartesian magnetic quadrupole moment tensor of a "figure-8" current loop (loop radius a) with current flow I as indicated in the diagram below? (There is no short-circuit at the cross-over point.) Note that a **single** circular loop does not have a quadrupole moment **about its center**. Hint: use the problem above.



BONUS (due when the homework is due):
What is the vector potential for a point magnetic dipole $\vec{m}$ at the origin? Show by calculation that the magnetic field obtained by taking the curl of this vector potential is the same as the magnetic field obtained by taking the negative gradient of the magnetic scalar potential in the first problem.