Physics 7312 — Professor Scalise
Homework Assignment #4 — Spring 2023

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1. A spherical shell of radius R, carrying a uniform surface charge density σ, spins with angular velocity $\omega\hat{z}$.
   (a) Find the vector potential everywhere in spherical polar coordinates.
   (b) Find the magnetic field inside the shell in Cartesian coordinates.

2. A solid sphere of radius a and total charge Q has a volume charge density given by

$$\rho(\vec{r}) = \begin{cases} \rho_0(1+\gamma\cos\theta) & , \; r \le a \\ 0 & , \; r > a \end{cases}$$

   where $\rho_0 = \dfrac{Q}{\frac{4}{3}\pi a^3}$ and γ is a dimensionless parameter. The solid sphere rotates about the polar axis (z-axis) with angular velocity $\vec{\omega} = \omega\hat{z}$.
   (a) Calculate the first two non-vanishing irreducible Cartesian magnetic multipole moment tensors.
   (b) Calculate the contribution to the magnetic scalar potential $\Phi_m(\vec{r})$ from these two moments in spherical polar coordinates.
   (c) Calculate the magnetic field $\vec{B}(\vec{r})$ arising from these two moments in spherical polar coordinates.

BONUS (due when the homework is due):
See the next page.

Physics Challenge for Teachers and Students

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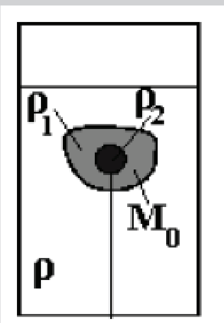
Hanging by a thread

One end of a light string is attached to a small aluminum ball inserted into a chunk of ice of mass $M_0 = 0.100$ kg. The other end of the string is attached to the bottom of an insulated container filled with $m = 0.500$ kg of water. Initially, the tension in the string is 0.080 N, the temperature of the water is 20 °C, and the temperature of the ice and the aluminum ball is 0 °C.

What is the temperature of the water at the moment the tension in the string becomes zero?

Assume that the thermal equilibrium within the water is established instantly and use the following reference information:

The specific heat of water is $c = 4200$ J/(kg × °C).
The heat of fusion of ice is $\lambda = 3.30 \times 10^5$ J/kg.
The density of water is $\rho = 1000$ kg/m^3.
The density of ice is $\rho_1 = 900$ kg/m^3.
The density of aluminum is $\rho_2 = 2700$ kg/m^3.



We received a relatively small number of correct solutions to our November Challenge, ***Pros and cones.**** We are pleased to recognize the following successful solvers:

Philip Blanco (Grossmont College, El Cajon, CA)

Phil Cahill (The SI Organization, Inc., Rosemont, PA)

Tom Dauer (EOS IT Solutions, Oakland, CA)

Norman Derby (Southwestern Oregon Community College, Brookings, OR)

Supriyo Ghosh (Kolkata, India)

José Ignacio Íñiguez de la Torre (Universidad de Salamanca, Salamanca, Spain)

Carl E. Mungan (U. S. Naval Academy, Annapolis, MD)

Krzysztof Rębilas (Uniwersytet Rolniczy im. Hugona Kołłątaja w Krakowie, Kraków, Poland)

Pascal Renault (John Tyler Community College, Midlothian, VA)

Randall J. Scalise (Southern Methodist University, Dallas, TX)

Quan Zheng (Indian River State College, Fort Pierce, FL)

Guidelines for contributors

- We ask that all solutions, preferably in Word format, be submitted to the dedicated email address ***challenges@aapt.org***. Each message will receive an automatic acknowledgment.
- If your name is—for instance—Taylor Swift, please name the file "***Swift23Feb***" (do not include your first initial) when submitting the February 2023 solution.
- The subject line of each message should be the same as the name of the solution file.
- The deadline for submitting the solutions is the last day of the corresponding month.
- Each month, a representative selection of the successful solvers' names will be published in print and on the web.
- If you have a message for the Column Editor, you may contact him at ***korsunbo@post.harvard.edu***; however, please do not send your solutions to this address.

Many thanks to all contributors and we hope to hear from many more of you in the future!

Note: we always welcome and appreciate reader-contributed original *Challenges!*

The solutions to the past *Challenges* can be found here: https://aapt.scitation.org/topic/collections/physics-challenge. Please note that AAPT membership may be required to view the files.

Boris Korsunsky, Column Editor

*Adapted from *The Problems of Moscow Physics Olympiads*, edited by M. Semenov and V. Yakuta (MCCME, Moscow, 2006).