Physics 7312
Homework Assignment #5

Professor Scalise
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1. (a) Find the interaction potential energy of two point magnetic dipoles $\vec{m}_{(1)}$ and $\vec{m}_{(2)}$ separated by $\vec{r} = \vec{r}_1 - \vec{r}_2$. Use $U = -\vec{m}_{(1)} \cdot \vec{B}_2(\vec{r}_1)$, where $\vec{B}_2(\vec{r}_1)$ is the magnetic field due to $\vec{m}_{(2)}$ at the position of the first point dipole $\vec{r}_1$.

   (b) What is U for the following configurations of the dipoles:

      i. $\uparrow\uparrow$

      ii. $\begin{matrix}\uparrow\\ \uparrow\end{matrix}$

      iii. $\uparrow\downarrow$

      iv. $\begin{matrix}\uparrow\\ \downarrow\end{matrix}$

      v. $\begin{matrix}\downarrow\\ \uparrow\end{matrix}$

2. The constant fields inside a magnetic material that fills all space are $\vec{B}_0$, $\vec{H}_0$, and $\vec{M}_0$ with $\vec{H}_0 = \frac{1}{\mu_0}\vec{B}_0 - \vec{M}_0$, all aligned along the z-direction.

   (a) Find the magnetic field and magnetic intensity inside a long thin needle-shaped cavity running parallel to $\vec{M}_0$.

   (b) Find the magnetic field and magnetic intensity inside a thin flat disk-shaped cavity with symmetry axis parallel to $\vec{M}_0$.

   (c) Find the magnetic field and magnetic intensity inside a small spherical cavity.

   Hint: Carving out a cavity is the same as superposing an object of the same shape but with opposite magnetization.

3. A magnetically hard material is in the shape of a right circular cylinder of length L and radius a. The cylinder has a permanent magnetization M_0 uniform throughout its volume and parallel to its axis.

   (a) Determine the B and H fields at all points on the axis of the cylinder, both inside and outside.

   (b) Plot (not by hand) the ratios $B/\mu_0 M_0$ and H/M_0 on the axis as functions of z for $L/a = 5$.