Physics 7312 Professor Scalise
Homework Assignment #8 Spring 2023

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1. Consider incident light which is described by a density matrix $\rho_{\rm inc}$. The density matrix of the reflected light is given by

$$\rho_{\rm refl} = R\, \rho_{\rm inc}\, R^{\dagger}$$

where R is a matrix that involves the reflection process, that is, it involves μ, μ', n, n', and the angle of incidence θ_i. Find R.

2. (a) What are the two angles of incidence at which unpolarized light reflected from a planar air-water interface will be completely polarized? Draw pictures. (The permeabilities of air and water are close to that of vacuum.) How are these two angles related to each other?

   (b) What is the direction of the polarization? Draw a picture. If you are standing on the shore of a calm lake looking at the reflected light of the sun, is the reflected light polarized horizontally or vertically?

   (c) What is the angle of incidence at which light is totally internally reflected from a planar air-water interface? Draw a picture.

3. Show that Maxwell's equations in free space are invariant under the following duality transformation:

$$\vec{E}' = \cos(\alpha)\vec{E} + \sin(\alpha)c\vec{B}, \qquad c\vec{B}' = -\sin(\alpha)\vec{E} + \cos(\alpha)c\vec{B}$$

where α is an arbitrary real angle; that is, show that $\vec{E}'$ and $\vec{B}'$ satisfy the same field equations as $\vec{E}$ and $\vec{B}$. Verify that the energy density of the electromagnetic field and the Poynting vector are also invariant under this transformation.

BONUS (due when the homework is due):
A non-reflective coating is applied at an air/glass interface. What are the index of refraction and the thickness of the coating?