Physics 7312 Professor Scalise
Homework Assignment #10 Spring 2023

1. Consider a charge q undergoing simple harmonic motion in the z-direction.

$$z(t) = z_0 \cos \omega t$$

(a) What is the current density $\vec{J}(\vec{r}, t)$ in Cartesian coordinates?

In the following, you may find the following expression for the Bessel function useful:

$$J_n(u) = i^n \oint \frac{d\phi}{2\pi} e^{in\phi} e^{-iu\cos\phi}$$

Because the system is periodic in time, use the Fourier **series** rather than the Fourier **transform** in the time variable. You still need the Fourier transform in the space variables.

(b) Find the Fourier coefficient $\vec{J}_{\vec{k}_n,\omega_n}$ and simplify the factor below that appears in the power formula:

$$\left[\vec{J}^*_{\vec{k}_n,\omega_n} \cdot \vec{J}_{\vec{k}_n,\omega_n} - \left(\hat{n} \cdot \vec{J}^*_{\vec{k}_n,\omega_n}\right)\left(\hat{n} \cdot \vec{J}_{\vec{k}_n,\omega_n}\right)\right]$$

BONUS (due when the homework is due):

See next page.

Physics Challenge for Teachers and Students

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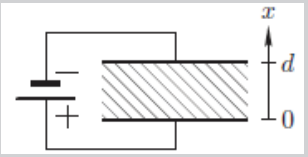
Resistivity is futile

A parallel-plate capacitor of capacitance C is connected to an ideal battery with emf V. The capacitor is filled with a low-conductivity material that has a dielectric constant 1 and the resistivity that varies as $\rho = \rho_0[1 + (2x/d)]$, where x is the distance from the bottom plate, and d is the plate separation, as shown in the diagram below.
Find the current through the capacitor and the energy stored in it.



We are pleased to recognize the following successful solvers of our January *Challenge*, ***Rolling uphill***.*

Phil Cahill (The SI Organization, Inc., Rosemont, PA)

Daniel Cartin (Naval Academy Preparatory School, Middetown, RI)

Norman Derby (Southwestern Oregon Community College, Brookings, OR)

Don Easton (Lacombe, Alberta, Canada)

Steven Morris (emeritus, Los Angeles Harbor College, Wilmington, CA)

Carl E. Mungan (U. S. Naval Academy, Annapolis, MD)

Francisco Pham (The Westwood School, Dallas, TX)

Pascal Renault (John Tyler Community College, Midlothian, VA)

Randall J. Scalise (Southern Methodist University, Dallas, TX)

Ahmet Uguz (Yahya Akel Science High School, Mersin, Turkey)

Quan Zheng (Indian River State College, Fort Pierce, FL)

Guidelines for contributors

- We ask that all solutions, preferably in Word format, be submitted to the dedicated email address ***challenges@aapt.org***. Each message will receive an automatic acknowledgment.
- If your name is—for instance—Sandra Magnus, please name the file "***Magnus23April***" (do not include your first initial) when submitting the April 2023 solution.
- The subject line of each message should be the same as the name of the solution file.
- The deadline for submitting the solutions is the last day of the corresponding month.
- Each month, a representative selection of the successful solvers' names will be published in print and on the web.
- If you have a message for the Column Editor, you may contact him at ***korsunbo@post.harvard.edu***; however, please do not send your solutions to this address.

Many thanks to all contributors and we hope to hear from many more of you in the future!

Note: we always welcome and appreciate reader-contributed original *Challenges!*

The solutions to the past *Challenges* can be found here: https://aapt.scitation.org/topic/collections/physics-challenge Please note that AAPT membership may be required to view the files.

Boris Korsunsky, Column Editor

*Adapted from *50 Physics Olympiad Problems* by A. P. Kuznetsov et al. (Nauchnaya Kniga, Saratov, 2006).