Physics 7314 Professor Scalise
Homework Assignment #4

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1. Consider the quantum harmonic oscillator. The energy eigenvalues are $E_n = (n+\frac{1}{2})\hbar\omega$. Recall that $\hat{a}$ is the lowering operatror and $\hat{a}^\dagger$ is the raising operator.

   (a) What is the unitary time evolution operator $\hat{U}(t,0)$ in terms of the Hamiltonian operator $\hat{H}$? (This is easy; don't complicate it.)

   (b) By calculating their action on energy eigenkets $|n\rangle$, find expressions for the time-dependent Heisenberg picture operators $\hat{B}_H(t)$ in terms of the time-independent Schrödinger picture operators $\hat{B}_S$:

      i. $\hat{a}_H(t) = \hat{U}^\dagger(t,0)\ \hat{a}_S\ \hat{U}(t,0)$
      ii. $\hat{a}^\dagger_H(t) = \hat{U}^\dagger(t,0)\ \hat{a}^\dagger_S\ \hat{U}(t,0)$
      iii. $\hat{X}_H(t) = \hat{U}^\dagger(t,0)\ \hat{X}_S\ \hat{U}(t,0)$
      iv. $\hat{P}_H(t) = \hat{U}^\dagger(t,0)\ \hat{P}_S\ \hat{U}(t,0)$

   (c) Show that $\hat{U}^\dagger(t=\frac{\pi}{2\omega},0)|x\rangle$ is an eigenket of the Schrödinger picture momentum operator $\hat{P}$ and find its eigenvalue.

2. Complex contour integration

   (a) Show that all the zeros of $\cos(z)$ are real. Hint: write $z = x+iy$ and use the angle addition formula, then equate the real and imaginary parts to zero and solve these two coupled equations.

   (b) Use the previous problem to evaluate $\oint_{|z|=2} \tan(z)dz$.