Physics 7314 — Professor Scalise

Homework Assignment #5

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1. (a) For the nth energy eigenstate $|n\rangle$ of the quantum harmonic oscillator, find $\sigma_x \sigma_p$ and check the uncertainty principle. (This is easier if you write x and p in terms of the raising and lowering operators.)

   (b) Now we are going to construct linear combinations of the energy eigenstates that saturate the inequality in the uncertainty relation. These "coherent states" happen to be eigenstates of the lowering operator $\hat{a}$.

   $$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

   where the eigenvalue α can be any complex number. Show that $\sigma_x \sigma_p = \hbar/2$.

   (c) Expand $|\alpha\rangle$ in terms of energy eigenstates

   $$|\alpha\rangle = \sum_{n=0}^{\infty} c_n |n\rangle$$

   and find the expansion coefficients c_n. Remember that the ket $|\alpha\rangle$ is normalized.

   (d) Next add in the time dependence

   $$|n\rangle \longrightarrow \exp(-iE_n t/\hbar)|n\rangle$$

   and show that $|\alpha(t)\rangle$ is still an eigenket of the lowering operator, but now the eigenvalue is time dependent $\alpha(t) = \exp(-i\omega t)\alpha$.

   (e) Is the ground state of the QHO, $|0\rangle$, a coherent state?