Physics 7314 Professor Scalise
Homework Assignment #6

1. The electromagnetic field strength tensor, called Faraday, is defined with two contravariant indices as

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \\ E^3 & -B^2 & B^1 & 0 \end{pmatrix}$$

   (a) What is $F_{\mu\nu}$ with two covariant indices? Does this depend on whether we use the particle or gravity metric tensor?

   (b) What is $F'^{\mu\nu}$ in a frame boosted along the x-axis by velocity $\vec{v}$? This will show that the electric and magnetic fields do not transform the way that the 3-vector parts of a 4-vector like x_μ would transform.

   (c) What is $F^\mu{}_\mu$?

   (d) What is $F^{\mu\nu}F_{\mu\nu}$?

   (e) What is the Hodge dual tensor $G^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$?

   (f) What is $F^{\mu\nu}G_{\mu\nu}$?