REVIEW OF SPECIAL RELATIVITY — DATE: 08/27/25

NOTABLY NOT GENERAL RELATIVITY, QFT IS DONE IN FLAT SPACETIME

$\Rightarrow$ FLAT SPACE TIME OR MINKOWSKI METRIC

NOTE: "SPECIAL RELATIVITY DOES NOT REQUIRE CONSTANT VELOCITY DESPITE WHAT THE TALKING HEADS SAY"

GREEK INDICIES $\mu, \nu, \sigma, \tau \Rightarrow [0, 1, 2, 3]$

LATIN INDICIES $i, j, k, l \Rightarrow [1, 2, 3]$

UPPER INDEX : x^μ IS KNOWN AS CONTRAVARIAN

LOWER INDEX : x_μ IS KNOWN AS COVARIANT

NOTE: COLEMAN USES NATURAL UNITS AS $\hbar = c = 1$

$[m] = [E] = [t]^{-1} = [L]^{-1} \Rightarrow$ MASS = ENERGY = $\frac{1}{\text{TIME}}$ = $\frac{1}{\text{LENGTH}}$

THE FOUR VECTOR FOR QFT FOR POSITION IS GIVEN BY:

$$x^\mu = (x^0 = ct,\ x^1 = x,\ x^2 = y,\ x^3 = z) = (ct, x, y, z)$$

$$= (ct, \vec{r}) = (ct, \vec{x}) = (t, \vec{x})$$

THE ENERGY-MOMENTUM FOUR VECTOR IS GIVEN BY:

$$p^\nu = (p^0 = E/c,\ p^1 = p_x,\ p^2 = p_y,\ p^3 = p_z) = (p^0, p^1, p^2, p^3)$$

$$= (p^0, \vec{p}) = (E/c, \vec{p}) = \underline{(\omega, \vec{k})}$$

ω = ANGULAR FREQUENCY

$\vec{k}$ = WAVE VECTOR

$p^\nu p_\nu = \pm m^2c^2 \Rightarrow$ REMEMBER $\vec{p} \neq m\vec{v}$, $\vec{p} = m\vec{v}\gamma$

PARTICLE PHYSICS USES THE FOLLOWING METRIC

$$g_{\mu\nu} = \begin{array}{c} \\ \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \begin{matrix} \nu=0 & \nu=1 & \nu=2 & \nu=3 \\ t & x & y & z \end{matrix} \end{array} \begin{array}{l} \mu \\ \mu = 0 \rightarrow t \\ \mu = 1 \rightarrow x \\ \mu = 2 \rightarrow y \\ \mu = 3 \rightarrow z \\ \\ \end{array} \Rightarrow \begin{array}{l} g_{00} = -1 \\ g_{11} = 1 \\ g_{22} = 1 \\ g_{33} = 1 \end{array}$$

THE METRIC $g_{\mu\nu}$ HAS SOME IMPORTANT PROPERTIES AS FOLLOWS:

$g_{\mu\nu} = g_{\nu\mu}$ $\therefore$ THE TENSOR IS SYMMETRIC.

$$g_{\mu\nu} = \begin{cases} g_{\mu\nu} = -1 & \text{WHEN } \mu = \nu = 0 \\ g_{\mu\nu} = +1 & \text{WHEN } \mu = \nu = 1, 2, 3 \\ g_{\mu\nu} = 0 & \text{WHEN } \mu \neq \nu \end{cases}$$

$g_{\mu\nu}$ IS SOMETIMES WRITTEN AS $\eta_{\mu\nu}$ DEPENDING ON THE TEXTBOOK.

4-VECTOR SCALAR DOT PRODUCT AND EINSTEIN SUMMATION CONVENTION

EINSTEIN SUMMATION CONVENTION: TWO AND ONLY TWO REPEATED INDICIES, ONE BEING UP (CONTRAVARIANT), ONE BEING DOWN (COVARIANT); IMPLIES AN INHERENT SUM OVER THE REPEATED INDICIES.

$$x \cdot y = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} x^{\mu} y^{\nu} g_{\mu\nu} = x^{\mu} g_{\mu\nu} y^{\nu} = \underline{x}\, \underline{\underline{g}}\, \underline{y}$$

$$= x^{\mu} y_{\mu} \equiv \sum_{\mu=0}^{3} x^{\mu} y_{\mu} = x^0 y_0 + x^1 y_1 + x^2 y_2 + x^3 y_3$$

$$x \cdot y = x^0 y^0 - x^1 y^1 - x^2 y^2 - x^3 y^3$$

COVARIANT COORDINATE VECTOR

$$x_{\mu} = \sum_{\nu=0}^{3} x^{\nu} g_{\nu\mu} = x^{\nu} g_{\nu\mu} = (x_0, x_1, x_2, x_3)$$

$$= (x^0, -x^1, -x^2, -x^3) = (t, -\vec{r}) = (t, -\vec{x})$$