REVIEW FROM PREVIOUS LECTURE

DATE: 9/17/25

OPERATORS IN THE HEISENBERG PICTURE CAN BE REPERSENTED BY THE FOLLOWING RELATIONSHIP:

$$\hat{O}_H(t) = \hat{U}^{+}(t,0)\,\hat{O}_s\,\hat{U}(t,0)$$

THE EVOLUTION (TIME) OF THE HEISENBERG OPERATORS

$$\frac{d\hat{O}_H}{dt} = \frac{dU^{+}}{dt}\hat{O}_s\hat{U}^{+} + \hat{U}^{+}\cancelto{0}{\frac{d\hat{O}_s}{dt}}\hat{U} + \hat{U}^{+}\hat{O}_s\frac{d\hat{U}}{dt}$$

* RECALL FROM LECTURE 5/6 $\frac{d\hat{U}}{dt} = \frac{1}{i\hbar}\hat{H}_s\hat{U}$, $\hat{H}_s$ IS THE HAMILTONIAN OPERATOR IN THE SCHRÖDINGER PICTURE.

* $\hat{U}^{+} = \exp\{+i\hat{H}_s t\}$, $\hat{H}_s = \hat{H}_s^{+}$, $[\hat{U}, \hat{H}] = 0$ OR $[\hat{U}^{+}, \hat{H}] = 0$

$$\frac{d\hat{O}_H}{dt} = \frac{1}{i\hbar}\left[-\hat{U}\hat{H}_s\hat{O}_s\hat{U} + \hat{U}\hat{O}_s\hat{H}_s\hat{U}\right]$$

$$= \frac{1}{i\hbar}\left[\hat{O}_H(t), \hat{H}_s\right] = \frac{d\hat{O}_H}{dt}$$

* KNOWN AS THE HEISENBERG PICTURE EQUATION OF MOTION

QUANTUM FIELD THEOREY MOTIVATION

A CONSEQUENCE OF QUANTUM MECHANICS IS TOO LOOK AT THE "SIMPILIEST" PROBLEM THE HYDROGEN ATOM

$$\underset{\infty\text{-DIM}}{\mathcal{H}_{Part}} \otimes \underset{2\text{-DIM}}{\mathcal{H}_{Spin}}$$, WHERE $\mathcal{H}$ IS THE HILBERT SPACE.

HILBERT SPACE FOR PARTICLE MECHANICS WITH, l, p, x, y, z.

ALL $\mathcal{H}_{Spin}$ OPERATORS COMMUTE WITH $\mathcal{H}_{Part}$ OPERATORS.

AT SOME POINT IN QUANTUM THEORY, YOU WILL RUN INTO THE MAGNETIC DIPOLE MOMENT OF THE MAGNETIC DIPOLE. WITH THIS KNOWLEDGE, YOU CAN FINALLY FIND THE RATIO OF THE MAGNETIC MOMENT TO ITS ANGULAR MOMENTUM.

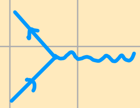
THIS RATIO IS KNOWN AS THE SPIN GYROMAGNETIC RATIO (g).

WHEN ADDING SPECIAL RELATIVITY TO THE QUANTUM MECHANICAL SCHRÖDINGER EQUATION, YOU GET THE KLEIN-GORDON OR THE DIRAC EQUATION.

* SPIN-0 * SPIN-1/2

THE CLASSICAL RESULT FOR THE GYROMAGNETIC RATIO OF THE ELECTRON (e^-), YEILDS $g_e = 2$. THIS VALUE IS MEASURED, IN THE MOST PRECISE EXPERIMENT ON THE PLANET.

* DIAGRAMS CONTAINING THE SIMPLEST VERTEX



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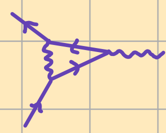


DIRAC EQUATION AT TREE LEVEL $\rightarrow g_e = 2$

DIRAC EQUATION WHEN LOOPS ARE INCLUDED $\rightarrow g_e \neq 2$

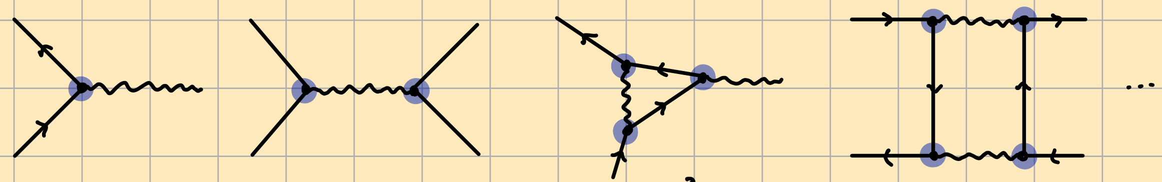
ANOMALOUS MAGNETIC DIPOLE MOMENT (a) :

$$a \equiv \frac{g-2}{2}$$

FOR (a) OF THE ELECTRON WE HAVE RELATIONSHIP TO α :

$$\alpha = \frac{e^2}{4\pi \varepsilon_0 \hbar c} \sim \frac{1}{137} \Rightarrow g_e = 2\left(1 + \frac{\alpha}{2\pi} + \cdots\right)$$

IMPORTANTLY α IS THE COUPLING CONSTANT IN QUANTUM ELECTRO DYNAMICS (QED).



α α^2 α^3 α^4

THIS COUPLING CONSTANT "RUNS" AS ENERGIES INCREASE, THE VALUE FOR a HAS A DEVIATION BETWEEN THEORY AND EXPERIMENT AS:

$a_{\text{THEO.}} = 0.001\ 159\ 181\ 643\ (764)$ - 5 LOOP CALCULATION BY TOM KINOSHITA

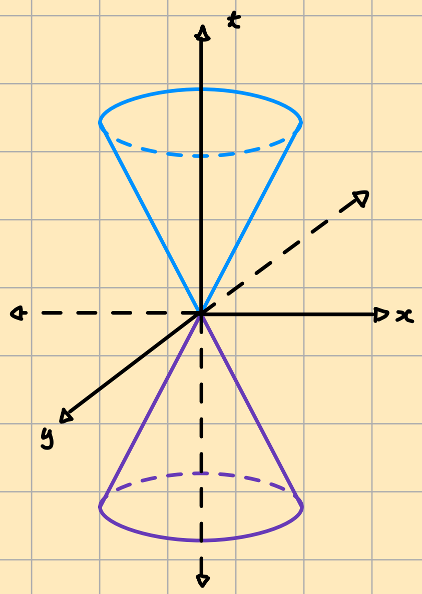
$a_{\text{EXP.}} = 0.001\ 159\ 180\ 59\ (13)$

FINALLY, THE DEATH OF QUANTUM MECHANICS:

START WITH A FREE PARTICLE OF MASS (m) SHARPLY LOCALIZED NEAR THE ORIGIN AT TIME (t) EQUALS ZERO.

$$\psi_0(x) = \langle \vec{x} | \psi_0 \rangle = \delta^{(3)}(\vec{x}) = \delta(x-0)\,\delta(y-0)\,\delta(z-0)$$

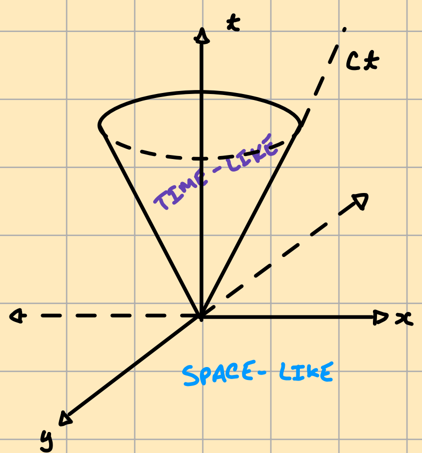
WHAT IS THE AMPLITUDE (A) FOR THE PARTICLE TO TRAVEL OUTSIDE ITS FUTURE LIGHTCONE?



* FUTURE LIGHT CONE

* PAST LIGHT CONE

LIGHT CONES ARE A PRINCIPLE FROM RELATIVITY. THEY REPRESENT THE PLACES A PARTICLE CAN TRAVEL TO WITHOUT EXCEEDING THE SPEED OF LIGHT c IN VELOCITY. THEY HAVE TWO DISTINCT REGIONS KNOWN AS SPACE-LIKE AND TIME-LIKE.



$$x^2 + y^2 + z^2 = c^2 t^2 \Rightarrow$$

$$x^2 + y^2 + z^2 = t^2 \Rightarrow$$

$$r^2 = t^2$$

* $r > ct$

* $r < ct$

THE PARTICLE'S KET $|\Phi\rangle$ IS LOCALIZED AT $\vec{r}(x, y, z)$ AT TIME t.

CALCULATING THE AMPLITUDE (A) YEILDS: * OUTSIDE OF THE LIGHT-CONE

$$A = \langle \Phi | e^{-i\hat{H}t} | \psi_0 \rangle \equiv \langle \vec{r} | e^{-i\hat{H}t} | \vec{0} \rangle$$

IF $A \neq 0$, THE THERE IS A NON ZERO PROBABILITY (A^*A) FOR THE PARTICLE TO TRAVEL FASTER THAN LIGHT. THIS RESULT, IS NOT EXPLAINABLE IN THE FRAMEWORK OF QUANTUM MECHANICS, THIS IS THE DEATH OF QUANTUM MECHANICS.