QUARKONIUM

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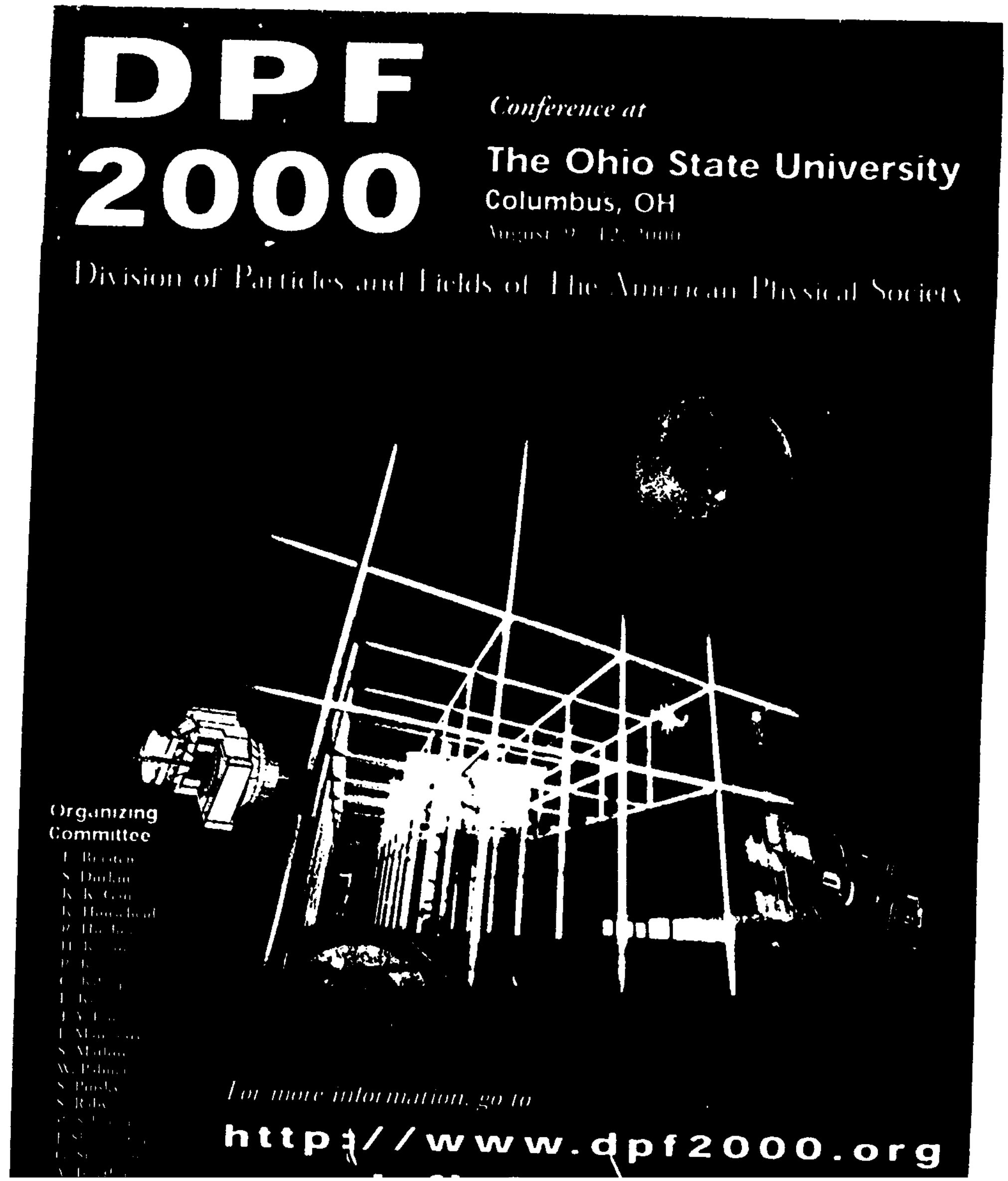
Lecture 1

What is quarkonium?

What is it good for?

Positronium

Quarkonium



ews on Quarkonium and NRQCD

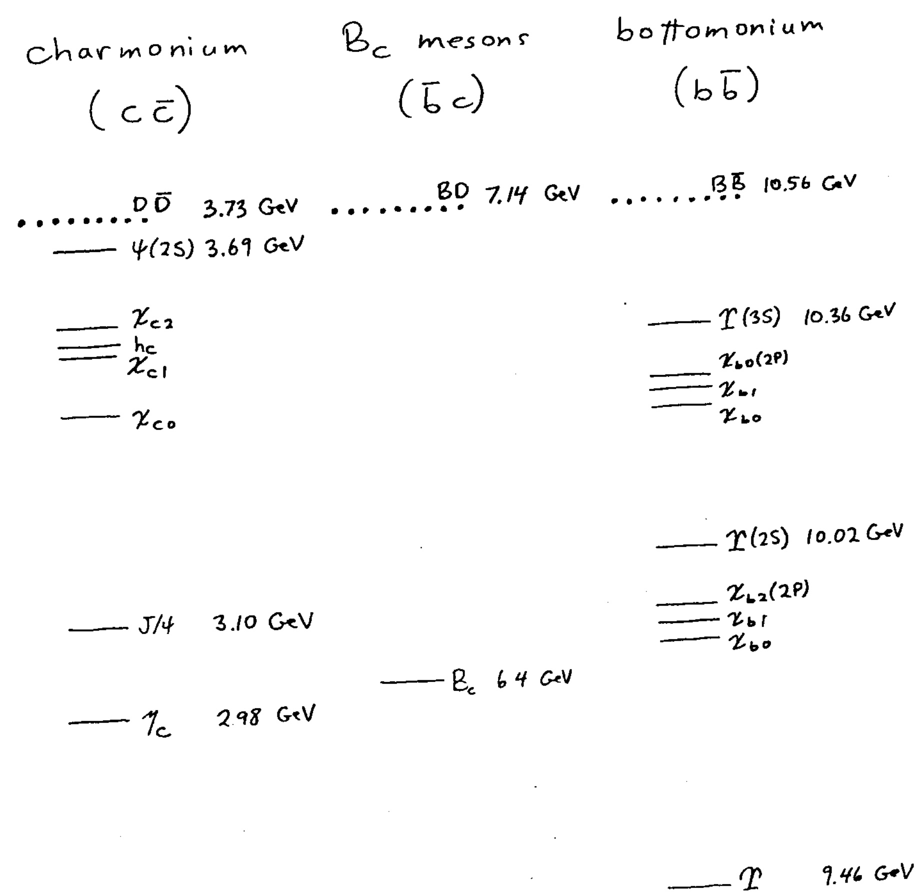
-aaten, S. Fleming, and T.C. Yuan,
›uction of Heavy Quarkonium
in High Energy Colliders
Rev. Nucl. Part. Sci. 46, 197 (1996).

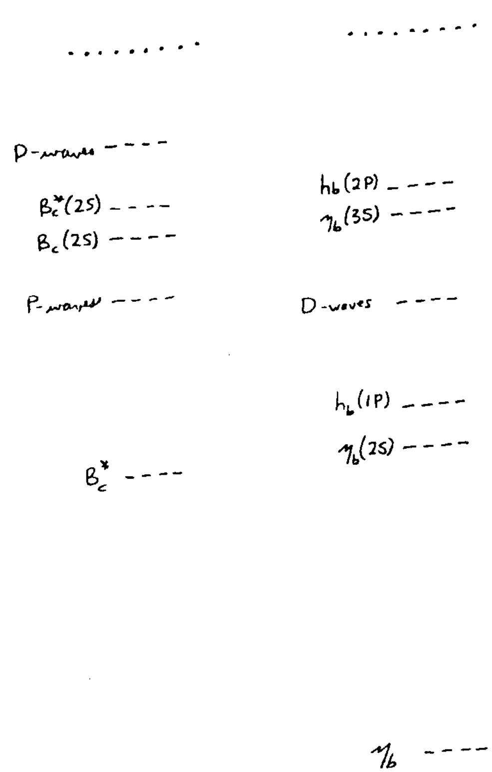
raaten,
roduction to the NRQCD Factorization
to Heavy Quarkonium
ph/9702225

ıeneke,
relativistic Effective Theory for Quarkonium
Production in Hadron Collisions
ph/9703429

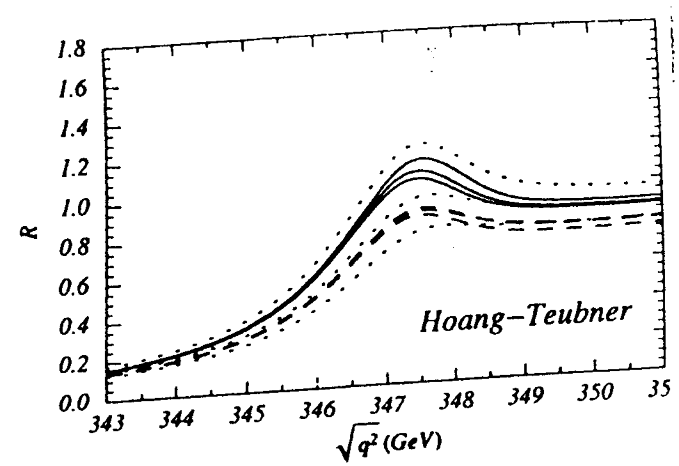
Rothstein,
QCD: A Critical Review
-ph/9911276

Quarkonium





• Hoang + Teubner, hep-ph/9904468



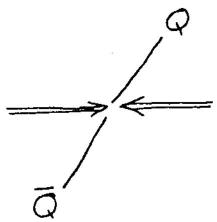
$$R = \frac{\sigma[e^+e^- \to t\bar{t} + X]}{\sigma[e^+e^- \to \mu^+\mu^-]}$$

!ARKONIUM

ɔrobe for heavy quarks

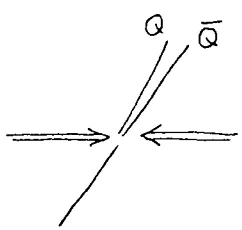
ɹy meson production

˃ production of $Q\bar{Q}$



ɹrkonium production

⇒ production of $Q\bar{Q}$
with small relative momentum



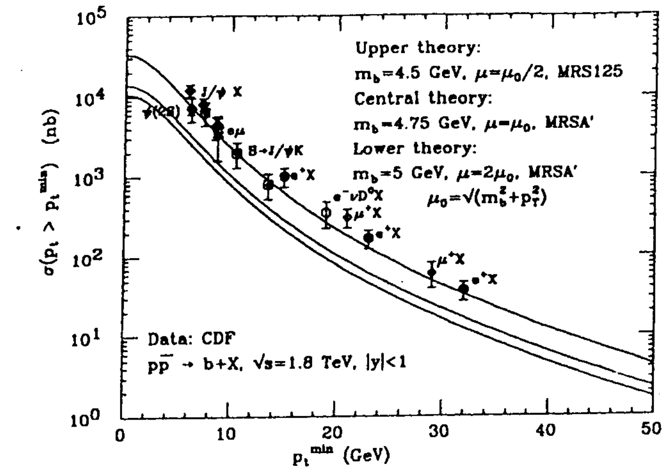


Figure 21. *CDF data on the integrated b-quark p_T distribution, compared to the results of NLO QCD.*

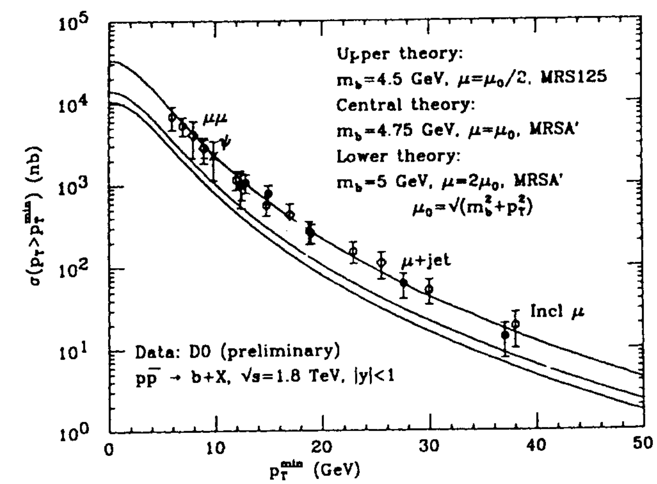
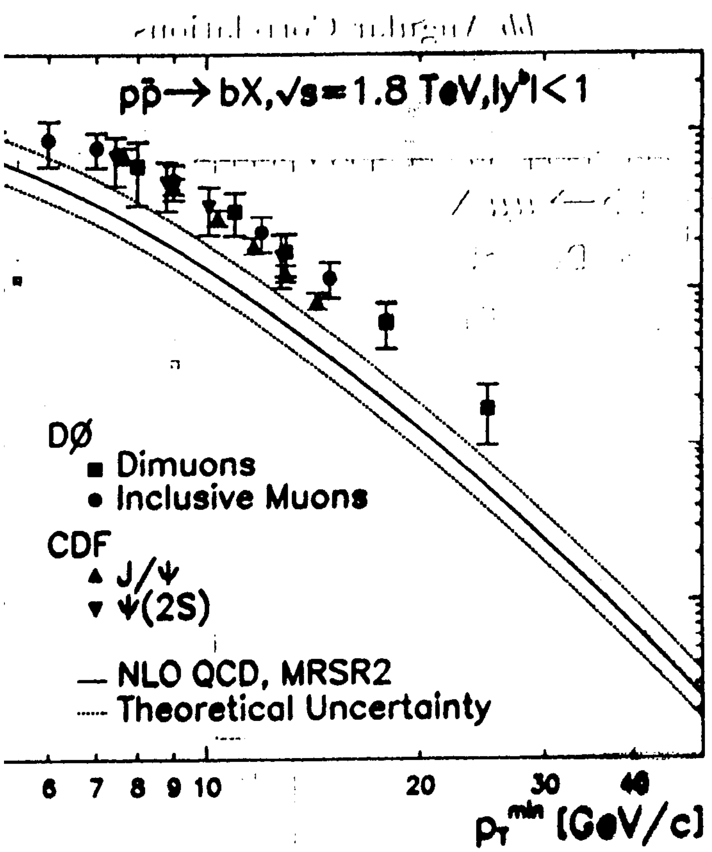


Figure 22: *D0 data on the integrated b-quark p_T distribution, compared*

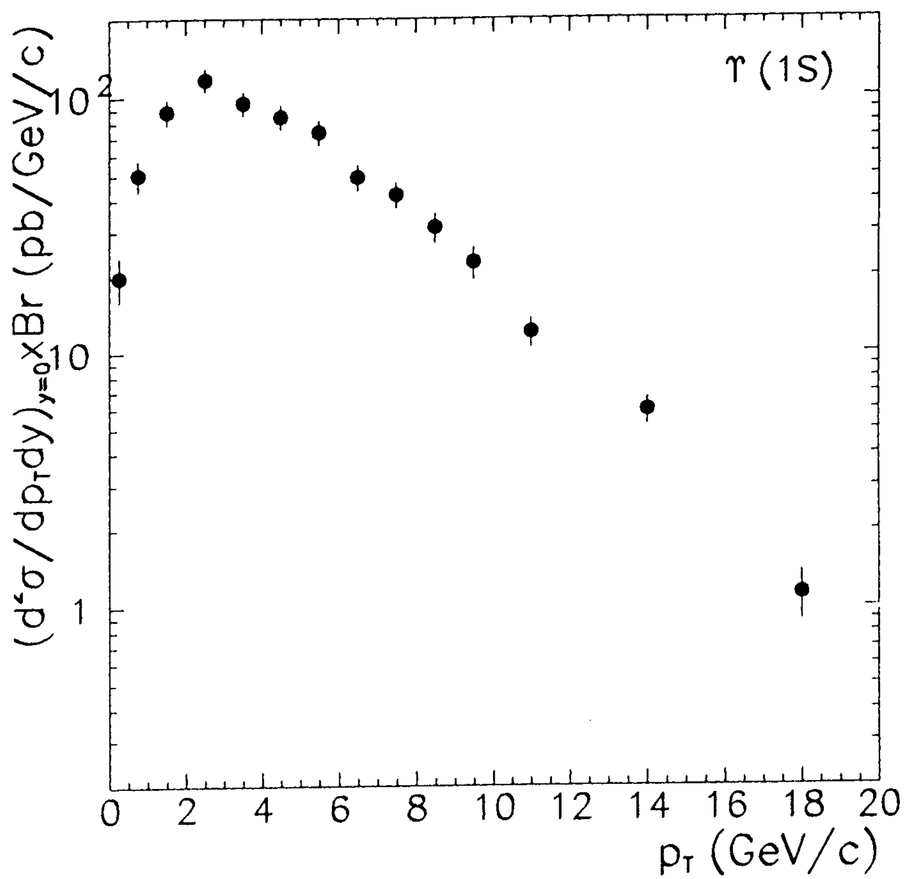
Frixione, Mangano, Nason, and Ridolfi, hep-ph/9702287

nclusive b-Quark Production Cross Sections



- NLO QCD predicts the shape of the cross section well, but underestimates its magnitude by a factor of three.
- DØ and CDF measurements in agreement.

CDF Preliminary



$B_{\Upsilon \to \mu^+\mu^-} \times \frac{d^2\sigma}{dp_T\, dy}\left[p\bar{p} \to \Upsilon + X\right]\Big|_{y=0}$ vs. p_T

UARKONIUM

flavor probe for jets ??

oft collinear dileptons"

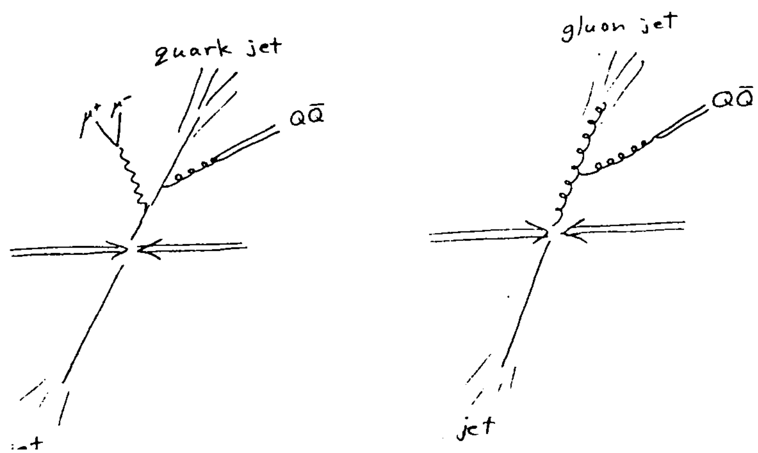
$\Longrightarrow$ photon radiation

$\Longrightarrow$ quark jet

oft collinear quarkonium

$\Longrightarrow$ gluon radiation

$\Longrightarrow$ gluon jet (or quark jet)



Coulomb problem

$$H_C = -\frac{1}{2m}\nabla^2 - \frac{\alpha}{r} \qquad m = m_e/2$$

$$\alpha \approx \frac{1}{137}$$

qualitative analysis

assume one important length scale R

$$E \sim \frac{1}{m}\left(\frac{1}{R}\right)^2 \sim \frac{\alpha}{R}$$

$$\Longrightarrow \quad R \sim \frac{1}{\alpha m}$$

$$E \sim \alpha^2 m$$

$$v \sim \alpha$$

exact solution

$$E_n = -\frac{\alpha^2 m}{2n^2} \qquad n = 1, 2, \ldots$$

$$\langle v^2 \rangle^{1/2} = \frac{\alpha}{n}$$

OSITRONIUM Fine structure

ngular momenta: orbital $\vec{L}$

spins $\vec{S} = \vec{S}_1 + \vec{S}_2$

total $\vec{J} = \vec{L} + \vec{S}$

amiltonian: $H = H_C + H_{rel} + H_{SO} + H_{SS}$

elativistic: $H_{rel} = -\frac{1}{8m^3}(\nabla^2)^2$

$$\Delta E \sim \frac{1}{m^3}\frac{1}{R^4} \sim \alpha^4 m$$

pin-orbit: $H_{SO} = \frac{\alpha}{2m^2}\frac{\vec{L}\cdot\vec{S}}{r^3}$

$$\Delta E \sim \frac{\alpha}{m^2}\frac{1}{R^3} \sim \alpha^4 m$$

pin-spin: $H_{SS} = \frac{\alpha}{m^2}\left\{ \frac{8\pi}{3}\vec{S}_1\cdot\vec{S}_2\, \delta^3(\vec{r}) + \cdots \right\}$

$$\Delta E \sim \frac{\alpha}{m^2}\frac{1}{R^3} \sim \alpha^4 m$$

$\Rightarrow$ Coulomb state n

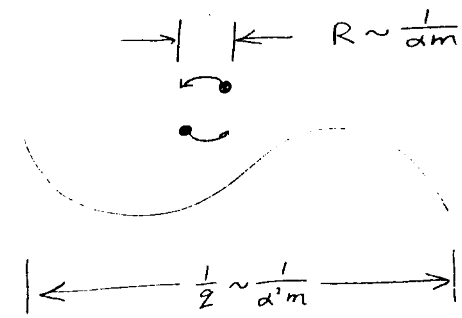
split into angular momentum states ${}^{2S+1}L_J$

POSITRONIUM: 3-body effects

contribution from $|e^+e^-\gamma\rangle$ state

with photon momentum $q \sim \alpha^2 m$

$$\Delta E \sim \alpha^4 m$$



		probability
$\vert\text{positronium}\rangle =$	$\vert e^+e^-\rangle$	~ 1
	$+ \vert e^+e^-\gamma\rangle$	$\sim \alpha^2$
	$+ \cdots$	

OSITRONIUM summary

nonrelativistic

$$v \sim \alpha$$

mostly two-body $|e^+e^-\rangle$

probability of $|e^+e^-\gamma\rangle \sim \alpha^2$

Energy splittings

radial: $\Delta E \sim \alpha^2 m_e$

orbital: $\sim \alpha^4 m_e$

spin: $\sim \alpha^4 m_e$

3-body: $\sim \alpha^4 m_e$

QUARKONIUM "potential model"

$$H = -\frac{1}{2M}\nabla^2 + V(r) \qquad M = M_Q/2$$

$$V(r) \approx -\frac{4}{3}\frac{\alpha_s(1/R)}{R} + K^2 R \qquad K = 0.45 \text{ GeV}$$

qualitative analysis:

assume one important length scale R

case 1: Coulomb term dominates ($t\bar{t}$, bb?)

$$E \sim \frac{1}{M}\left(\frac{1}{R}\right)^2 \sim \frac{\alpha_s(1/R)}{R}$$

$$\Longrightarrow \quad R \sim \frac{1}{\alpha_s(1/R)\, M}$$

$$E \sim M v^2$$

$$v \sim \alpha_s(Mv)$$

UARKONIUM "potential model"

$= -\frac{1}{2M}\nabla^2 + V(r)$ $\qquad M = M_Q$

$r) \approx -\frac{4}{3}\frac{\alpha_s(1/r)}{r} + K^2 r$

---

ase 2: <u>linear</u> term dominates $(c\bar{c},\ bb?)$

$$E \sim \frac{1}{M}\left(\frac{1}{R}\right)^2 \sim K^2 R$$

$$\Longrightarrow R \sim (K^2 M)^{-1/3}$$

$$E \sim M v^2$$

$$v \sim \left(\frac{K}{M}\right)^{2/3}$$

---

alance between kinetic and potential energy

$\Longrightarrow$ small parameter v

$$v \sim \alpha_s(Mv) \quad \text{or} \quad v \sim \left(\frac{K}{M}\right)^{2/3}$$

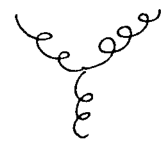
Scales in Quarkonium

	Mv^2	Mv	M		
$c\bar{c}$	0.5 GeV	0.9 GeV	1.5 GeV	$\Longrightarrow$	$v^2 \approx \frac{1}{3}$
$b\bar{b}$	0.5	1.5	4.7		$\frac{1}{10}$
$t\bar{t}$	1.5	16	180		$\frac{1}{120}$

Running Coupling Constant

	$\alpha_s(Mv^2)$	$\alpha_s(Mv)$	$\alpha_s(M)$
$c\bar{c}$	—	0.52	0.35
$b\bar{b}$	—	0.35	0.22
$t\bar{t}$	0.35	0.16	0.11

n exploit small number v!

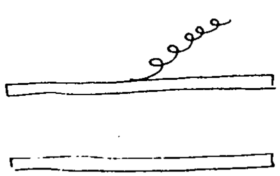


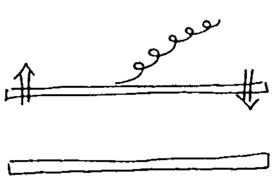
't gluons ($E \sim Mv^2$)

are strongly interacting

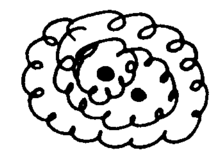
$\alpha_s(E) \gtrsim 1$

- emission of gluons that change angular momentum state of $Q\bar{Q}$ is suppressed by powers of v!

L  $L \pm 1$ amplitude $\sim v$

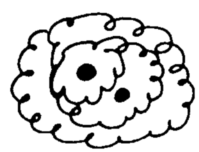
$= 1$  $S = 0$ amplitude $\sim v^2$

Structure of J/ψ (in Coulomb gauge)

	$c\bar{c}_1(^3S_1)$	$c\bar{c}_8(^3P_J) + X_g$
		
Probability	$P \sim 1$	$P \sim v^2$
Size	$R \sim \frac{1}{Mv}$	$R \sim \frac{1}{Mv^2}$
Binding energy	$E \sim Mv^2$	$\Delta E \sim Mv^4$

Structure of χ_{cJ}, $J=0,1,2$

(in Coulomb gauge)

$c\bar{c}_1(^3P_J)$	$c\bar{c}_8(^3S_1) + X_g$
	
$P \sim 1$	$P \sim v^2$
$R \sim \frac{1}{Mv}$	$R \sim \frac{1}{Mv^2}$
$E \sim Mv^2$	$\Delta E \sim Mv^4$

QUARKONIUM summary

- nonrelativistic

  $v \sim \alpha_s(Mv)$ or $\left(\frac{K}{M}\right)^{2/3}$

- mostly 2-body $|Q\bar{Q}\rangle$

  probability of $|Q\bar{Q}+X_g\rangle \sim v^2$

Energy splittings

radial: $\Delta E \sim Mv^2$

orbital: $\sim Mv^2$

spin: $\sim Mv^4$

3-body: $\sim Mv^4$

UARKONIUM

Eric Braaten
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ecture 2

- Color-singlet model
- Color-evaporation model
- NRQCD
- Annihilation decays
- Inclusive production

Quarkonium Production

Color-Singlet Model

Color Evaporation Model

NRQCD Factorization

Bodwin, Braaten, + Lepage (1995)

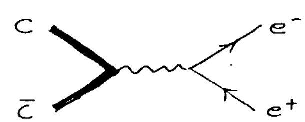
Color-Singlet Model for decays

$|J/\psi\rangle \approx |c\bar{c}_1(^3S_1)\rangle$

decay into e^+e^-:

$\Gamma[J/\psi \to e^+e^-]$

$\approx |\psi(0)|^2 \times \Gamma[c\bar{c}_1(^3S_1) \to e^+e^-]$

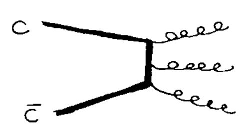


decay into light hadrons:

$\Gamma[J/\psi \to \text{light hadrons}]$

$\approx \Gamma[J/\psi \to \text{partons}]$

$\approx |\psi(0)|^2 \times \hat{\Gamma}[c\bar{c}_1(^3S_1) \to ggg]$



1 phenomenological parameter: $|\psi(0)|^2$

Color-Singlet Model for production

$|J/\psi\rangle \approx |c\bar{c}_1(^3S_1)\rangle$

inclusive production

$\sigma[J/\psi] \approx \sigma[c\bar{c}_1(^3S_1)] \times |\psi(0)|^2$

no phenomenological parameters!

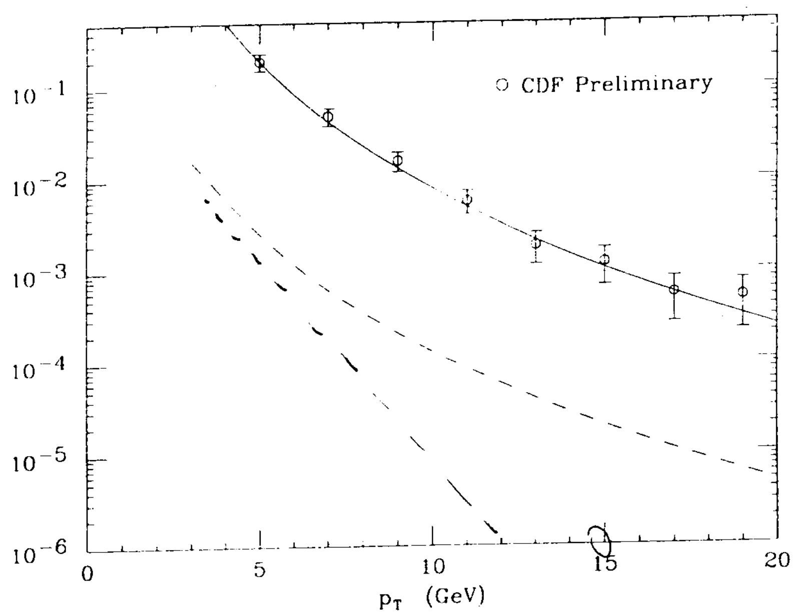
predictions

σ depends on spin m_S of H

ratios $\frac{\sigma[H]}{\sigma[J/\psi]}$ depend on process

...... CSM leading order (α_s^3)

— — — CSM fragmentation (α_s^5)



<u>Color-Evaporation Model</u> for production

e^+e^- production ?

<u>inclusive hadronic production</u>

$$\sigma[J/\psi] \approx \sigma[c\bar{c} : s < 4m_D^2] \times f_{J/\psi}$$

1 phenomenological parameter f_H
for each state H

<u>predictions</u>

σ independent of spin m_s of H

ratios $\dfrac{\sigma[H]}{\sigma[J/\psi]} = \dfrac{f_H}{f_{J/\psi}}$ independent of process

CEM
1975-2000
RIP

E-866: 800 GeV protons on Cu

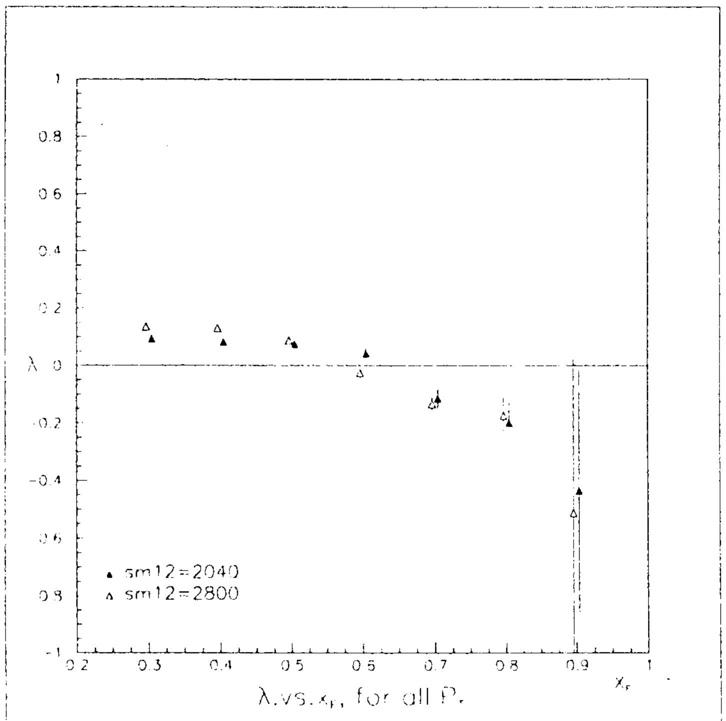


Figure 5.5: J/ψ polarization parameter λ in x_F bins. The errors are statistical only.

Quarkonium

$\approx$ nonrelativistic bound state of $Q\bar{Q}$

described by QCD

in terms of 2 parameters:

$\alpha_s \quad M_Q$

Important energy scales: M_Q

$M_Q v$

$M_Q v^2$

Λ_{QCD}

Small number v generated dynamically

$v \sim \alpha_s(M_Q v)$ or $v \sim \left(\frac{\kappa}{M_Q}\right)^{2/3}$

ales in Quarkonium

Mv^2	Mv	M		v^2
0.5 GeV	0.9 GeV	1.5 GeV	$\rightarrow$	$v^2 \approx \frac{1}{3}$
0.5	1.5	4.7		$\frac{1}{10}$
1.5	16	180		$\frac{1}{120}$

ning Coupling Constant

$\alpha_s(Mv^2)$	$\alpha_s(Mv)$	$\alpha_s(M)$
—	0.52	0.35
—	0.35	0.22
0.35	0.16	0.11

Large quark mass M $\Longrightarrow$

1. can use perturbative QCD at scale M
2. small number v generated dynamically

<u>Problems</u>

1. how to separate M from Mv, Mv^2, Λ_{QCD}
2. how to exploit $v^2 << 1$ for nonperturbative effects

<u>Solution</u> Lepage

NRQCD = formulation of QCD with NonRelativistic heavy quarks

QCD

$$\mathcal{L} = \mathcal{L}_{light} + \bar{\Psi}(i\gamma^\mu D_\mu - M)\psi$$

- relativistic heavy quarks
- conservation of $N_Q - N_{\bar{Q}}$

onrelativistic QCD

n effective field theory with

- nonrelativistic heavy quarks
- conservation of and

hat reproduces QCD

at momenta $\lesssim Mv$

to a specified order in v^2

NRQCD Lagrangian

$$\mathcal{L}_{NRQCD} = \mathcal{L}_{light} + \mathcal{L}_0 + \mathcal{L}_2 + \cdots$$

minimal lagrangian

$$\mathcal{L}_0 = \psi^\dagger\left(iD_0 + \frac{\mathbf{D}^2}{2M}\right)\psi + \chi^\dagger\left(iD_0 - \frac{\mathbf{D}^2}{2M}\right)\chi$$

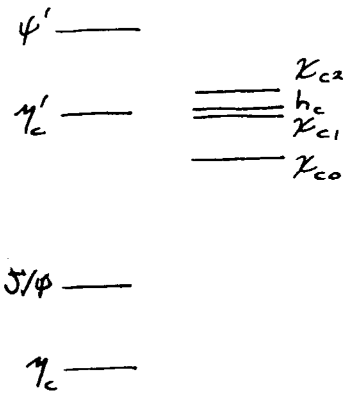
Spin symmetry !

v^2-improvement terms

$$\begin{aligned}\mathcal{L}_2 = \; & \frac{c_1}{8M^3}\psi^\dagger(\mathbf{D}^2)^2\psi + \frac{c_2}{8M^2}\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi \\ & + \frac{c_3}{8M^2}\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\sigma\psi + \frac{c_4}{2M}\psi^\dagger(g\mathbf{B}\cdot\sigma)\psi \\ & + \text{charge conjugate terms,}\end{aligned}$$

where $c_i = 1 + \mathcal{O}(\alpha_s)$

_arkonium Spectrum



n be calculated

ing Monte Carlo simulations

lattice NRQCD

$= \mathcal{L}_{light} + \mathcal{L}_0 \Longrightarrow$

– spin-averaged splittings to accuracy v^2

– no spin splittings

$= \mathcal{L}_{light} + \mathcal{L}_0 + \underline{\underline{\mathcal{L}_2}} \longrightarrow$

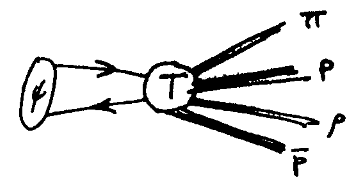
– spin-averaged splittings to accuracy v^4

– spin splittings to accuracy v^2

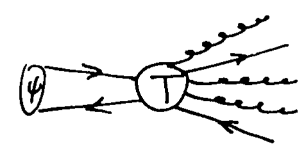
r higher accuracy, add $\mathcal{L}_4, \mathcal{L}_6, \ldots$

Factorization of Decay Rate

$$\Gamma = \sum_{\text{hadrons}} \left| \psi_{Q\bar{Q}} \otimes T_{Q\bar{Q} \to \text{hadrons}} \right|^2$$



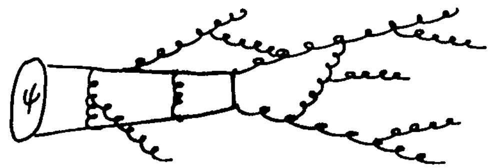
$$= \sum_{\text{partons}} \left| \psi_{Q\bar{Q}} \otimes T_{Q\bar{Q} \to \text{partons}} \right|^2$$



sum over hard partons (H)
soft partons (S)

$$= \sum_S \sum_H \left| \psi_{Q\bar{Q}} \otimes T_{Q\bar{Q} \to H+S} \right|^2$$

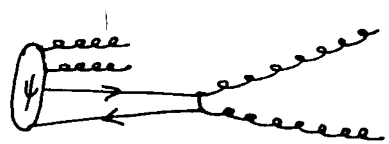
$$\Gamma = \sum_S \sum_H \left| \psi_{Q\bar{Q}} \otimes T_{Q\bar{Q} \to H+S} \right|^2$$



soft partons connected to H cancel

" $Q\bar{Q}$ can be absorbed into ψ

$$= \sum_S \sum_H \left| \psi_{Q\bar{Q}+S} \otimes T_{Q\bar{Q} \to H} \right|^2$$



$$= \sum_n \sum_S \left| \psi_{Q\bar{Q}[n]+S} \right|^2 \sum_H \left| T_{Q\bar{Q}[n] \to H} \right|^2$$

$$= \sum_n \langle \mathcal{O}_n \rangle \; \hat{\Gamma}_n$$

NRQCD Factorization Formula

for decay into light hadrons

$$\Gamma(\eta_c) = \sum_n \hat{\Gamma}(c\bar{c}_n) \langle \eta_c | \mathcal{O}_n | \eta_c \rangle$$

sum over all color/spin/orbital $c\bar{c}$

$$c\bar{c}_n = c\bar{c}_1(^{2S+1}L_J),\ c\bar{c}_8(^{2S}$$

short-distance factor

$\hat{\Gamma}(c\bar{c}_n) \propto$ annihilation rate for $c\bar{c}$ at threshold in state n

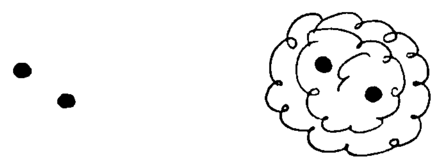
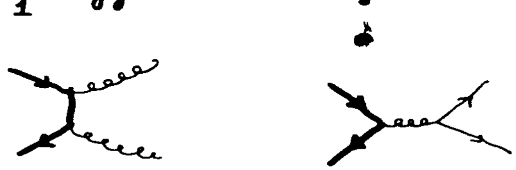
= power series in $\alpha_s(m_c)$

long-distance factor "NRQCD matrix elemen

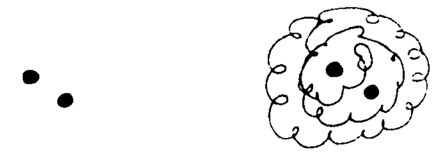
$\langle \,\cdot\, | \mathcal{O}_n | \,\cdot\, \rangle \propto$ probability for $\cdot$ to hav $c\bar{c}$ at same point in sta

– scales as v^{3+P_n}

ecay of η_c into light hadrons

	$c\bar{c}_1({}^1S_0)$	$c\bar{c}_8({}^1P_1) + X_g$
		
on l	$c\bar{c}_1 \to gg$	$c\bar{c}_8 \to q\bar{q}$
		
tance tor	$\hat{\Gamma} \sim \alpha_s^2$	$\hat{\Gamma} \sim \alpha_s^2$
ance or	$\langle O \rangle \sim v^3$	$\langle O \rangle \sim v^4 \times v^3$
e	$\Gamma \sim \alpha_s^2 v^3$	$\Gamma \sim \alpha_s^2 v^7$

Decay of χ_{cJ} into light hadrons

	$c\bar{c}_1({}^3P_J)$	$c\bar{c}_8({}^3S_1) + X_g$
		
Annihilation channel	$c\bar{c}_1 \to gg \ (J \neq 1)$	$c\bar{c}_8 \to q\bar{q}$
		
Short-distance factor	$\hat{\Gamma} \sim \alpha_s^2 \quad J=0,2$ $\sim \alpha_s^3 \quad J=1$	$\hat{\Gamma} \sim \alpha_s^2$
Long-distance factor	$\langle O \rangle \sim v^2 \times v^3$	$\langle O \rangle \sim v^2 \times v^3$
Decay rate	$\Gamma \sim \alpha_s^2 v^5 \quad J=0,2$ $\sim \alpha_s^3 v^5 \quad J=1$	$\Gamma \sim \alpha_s^2 v^5$

wave Decay Rates

at order $\alpha_s^2 v^5$

$$\text{c)} \approx \frac{5\pi\alpha_s^2(m_c)}{6m_c^2}\langle\mathcal{O}_8\rangle$$

$$\text{0)} \approx \frac{4\pi\alpha_s^2(m_c)}{m_c^4}\langle\mathcal{O}_1\rangle + \frac{n_f\pi\alpha_s^2(m_c)}{3m_c^2}\langle\mathcal{O}_8\rangle \quad \approx 14 \text{ MeV}$$

$$\text{1)} \approx \frac{n_f\pi\alpha_s^2(m_c)}{3m_c^2}\langle\mathcal{O}_8\rangle \quad \approx 0.64 \text{ MeV}$$

$$\text{c2)} \approx \frac{16\pi\alpha_s^2(m_c)}{45m_c^4}\langle\mathcal{O}_1\rangle + \frac{n_f\pi\alpha_s^2(m_c)}{3m_c^2}\langle\mathcal{O}_8\rangle \quad \approx 1.7 \text{ MeV}$$

ong-distance factors

$\langle \mathcal{O}_1 \rangle \propto |\nabla\psi(0)|^2$

$\psi(\vec{r})$ = wavefunction for $|c\bar{c}_1(^3P_J)\rangle$

$\langle \mathcal{O}_8 \rangle \approx |\psi_8(0)|^2$

$\psi_8(\vec{r})$ = "wavefunction for $|c\bar{c}_8(^3S_1)+X_g\rangle$

Annihilation Decay Rates – summary

NRQCD factorization formula

$$\Gamma[H] = \sum_n \langle H|\mathcal{O}_n|H\rangle\, \hat{\Gamma}[Q\bar{Q}_n]$$

S-waves

Γ is dominated by $|QQ_1\rangle$

$|Q\bar{Q}_8 + X_g\rangle$ suppressed by v^4

P-waves

$|Q\bar{Q}_1\rangle$ and $|Q\bar{Q}_8 + X_g\rangle$

both contribute at leading order in

clusive Production of charmonium

eation of $c\bar{c}$ pair
involves short distances $\lesssim \frac{1}{m}$

mation of charmonium
involves long distances $\gtrsim \frac{1}{mv}$

imming over final states
allow factoriz-ation methods of perturbative QCD
to be used to separate short distances
and long distances.

ort-distance factors are parton cross sections
calculable using perturbative QCD.

ng-distance factors
can be expressed as NRQCD matrix elements
that scale with definite powers of v^2.

Factorization $\equiv$ separation of scales

Scales in charmonium production include

$\sqrt{s}$ center-of-mass energy

p_T – transverse momentum of charmonium

m – mass of charm quark

mv – typical momentum of quark in charmonium

mv^2 – typical kinetic energy of quark in charmonium

Λ_{QCD} – scale of nonperturbative effects in light hadrons

Large quark mass M $\Longrightarrow$

1. can use perturbative QCD at scale M
2. small number v generated dynamically

Problems

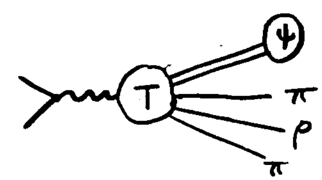
1. how to separate M from Mv, Mv^2, Λ_{QCD}
2. how to exploit $v^2 << 1$ for nonperturbative effects

Solution Bodwin, Braaten, + Lepage (1995)

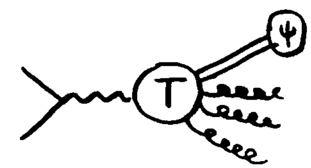
use NRQCD!

Factorization of Cross Section in e^+e^- annihilation

$$\sigma = \sum_{\text{hadrons}} \left| T_{Q\bar{Q}+\text{hadrons}} \otimes \psi_{Q\bar{Q}} \right|^2$$



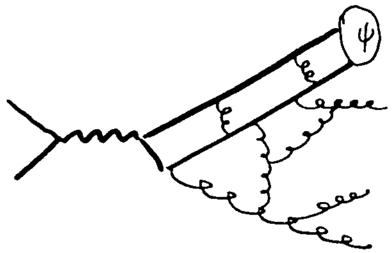
$$= \sum_{\text{partons}} \left| T_{Q\bar{Q}+\text{partons}} \otimes \psi_{Q\bar{Q}} \right|^2$$



sum over soft partons S in $Q\bar{Q}$ rest f

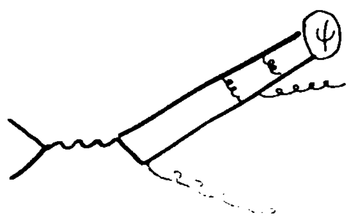
$$= \sum_{S} \left| T_{Q\bar{Q}+\ +S} \otimes \psi_{Q\bar{Q}} \right|^2$$

$$= \sum_S \sum_H \left| T_{Q\bar{Q}+H+S} \otimes \psi_{Q\bar{Q}} \right|^2$$



...t partons **connected to** H cancel

$$= \sum_S \sum_H \left| T_{Q\bar{Q}+H} \otimes A_{Q\bar{Q} \to Q\bar{Q}+S} \otimes \psi_{Q\bar{Q}} \right|^2$$



$$= \sum_n \quad \sum_S \left| A_{Q\bar{Q}_n \to Q\bar{Q}+S} \otimes \psi_{Q\bar{Q}} \right|^2$$

$$= \sum_n \quad \langle \mathcal{O}_n \rangle$$

Complications in production
with <u>0 hadrons</u> in initial state

In some regions of 4-momentum P of J/ψ,
derivation of <u>factorization breaks down</u>
because of <u>kinematic constraints</u>.

<u>Resonance region</u> $3\text{ GeV} < \sqrt{s} < 4\text{ GeV}$

problem: insufficient smearing over light hadron resonances

$$\Longrightarrow \sum_{\text{hadrons}} d\sigma \neq \sum_{\text{partons}} d\sigma$$

solution: smear over CM energy $\sqrt{s}$

<u>Endpoint region</u>: $E_{J/\psi} \longrightarrow \dfrac{\sqrt{s} + M_{J/\psi}^2}{2\sqrt{s}}$

$c\bar{c}$ recoils against hard gluon:

real soft gluons suppressed by kinematics
virtual " not suppressed
$\Longrightarrow$ cancellations incomplete

Complications in production
with <u>1 hadron</u> in the initial state

- factorization is only established for $p_T \gg \Lambda_{QCD}$
- at order $\frac{\Lambda_{QCD}^2}{p_T^2}$ (or higher),
  need correlation functions for
  for 2 (or more) partons in the hadron
- Is there factorization for $d\sigma$ integrated over p_T?

Factorization in m (rather than p_T)
is plausible, but not yet established.

error: $\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m^2}\right)$? $\mathcal{O}\left(\frac{(mv)^2}{m^2}\right)$?

Complications of production
with <u>2 hadrons</u> in the initial state

- **Smearing of parton p_T**
  from multiple soft gluon radiation
  may be important
- At order $\frac{\Lambda_{QCD}^4}{p_T^4}$,
  factorization breaks down completely
  (long range correlations between
  the partons in the 2 hadrons.)
- **Soft gluons connecting charmonium**
  to initial hadrons

  Qiu + Sterman?

RQCD Factorization Formula

for inclusive production

$$\sigma(J/\psi) = \sum_n \hat{\sigma}(c\bar{c}_n)\,\langle \mathcal{O}_n^{J/\psi} \rangle$$

sum over all color/spin/orbital $c\bar{c}$ state

$c\bar{c}_n = c\bar{c}_1({}^{2S+1}L_J),\ c\bar{c}_8({}^{2S+1}L_J)$

Short-distance factor

$\hat{\sigma}(c\bar{c}_n) \propto$ production rate of $c\bar{c}$ at threshold in state n

$=$ power series in $\alpha_s(m_c)$

Long-distance factor

$\langle \mathcal{O}_n \rangle \propto$ probability for pointlike $c\bar{c}$ pair in state n to form

scales like v^{3+P_n}

most important terms depend on ...

... magnitude of $d\hat{\sigma}$

- order in α_s
- color factors : e.g. $N_c^2 - 1 = 8$
- dependence on ratios of kinematic variables : e.g. m_c/p_T

... magnitude of $\langle \mathcal{O} \rangle$

leading matrix elements

for J/ψ : $\langle \mathcal{O}_1({}^3S_1) \rangle \sim 1$

$\langle \mathcal{O}_8({}^3S_1) \rangle,\ \langle \mathcal{O}_8({}^3P_J) \rangle,\ \langle \mathcal{O}_8({}^1S_0) \rangle \sim v^4$

for χ_{cJ} : $\langle \mathcal{O}_1({}^3P_J) \rangle \sim v^2$

$\langle \mathcal{O}_8({}^3S_1) \rangle \sim v$

Inclusive Production – summary

NRQCD factorization formula

$$\sigma = \sum_n \hat{\sigma}_n \langle \mathcal{O}_n \rangle$$

"NRQCD model" for ...

~~"Color-octet model"~~

... J/ψ production: $\langle \mathcal{O}_1(^3S_1) \rangle$

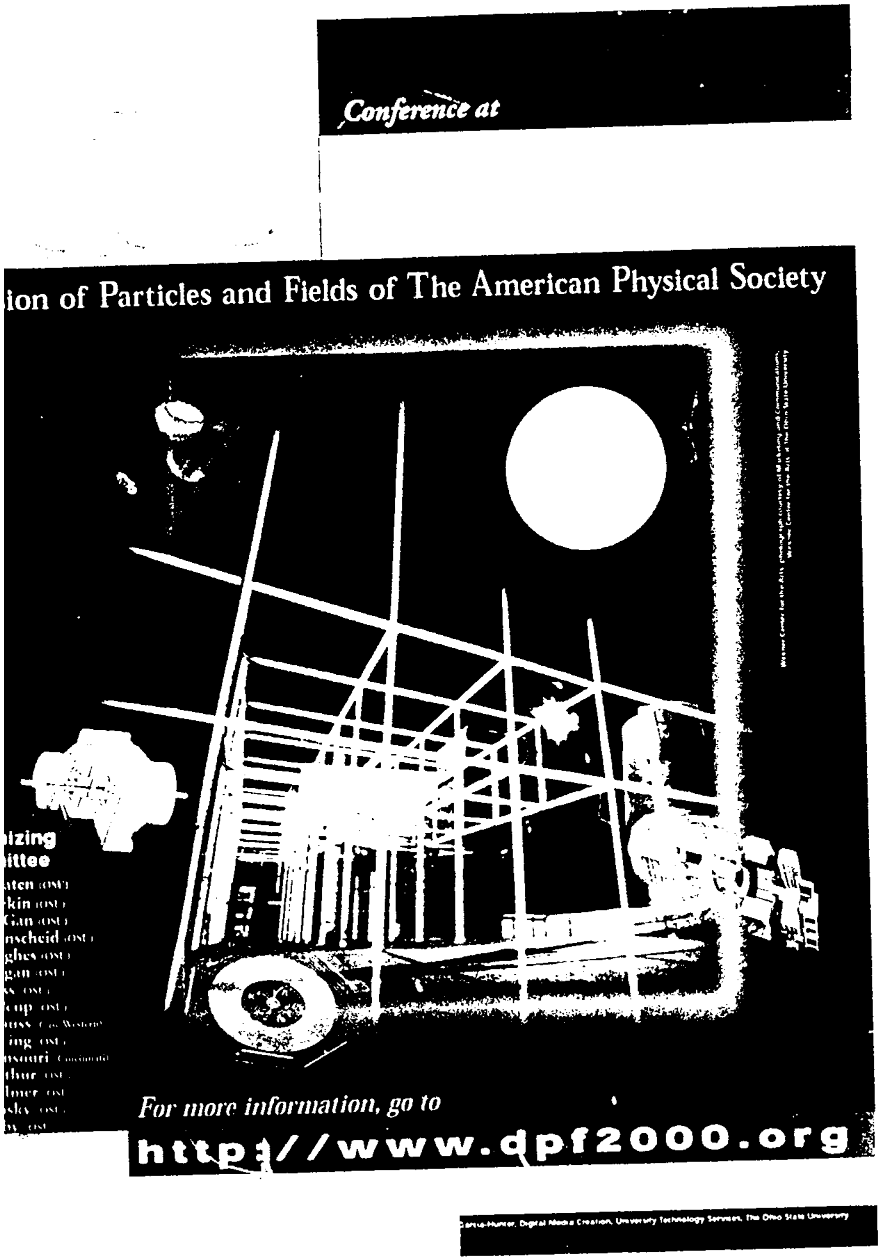
$\langle \mathcal{O}_8(^3S_1) \rangle$

$\langle \mathcal{O}_8(^1S_0) \rangle$

$\langle \mathcal{O}_8(^3P_0) \rangle$

... χ_{cJ} production: $\langle \mathcal{O}_1(^3P_J) \rangle$

$\langle \mathcal{O}_8(^3S_1) \rangle$



QUARKONIUM

Eric Braaten

Ohio State University

Lecture 3

Charmonium production at the Tevatron

- moderate p_T
- small p_T
- large p_T
- polarization

Inclusive Production – summary

NRQCD factorization formula

$$\sigma = \sum_n \hat{\sigma}_n \langle \mathcal{O}_n \rangle$$

"NRQCD model" for ...

~~"Color-octet model"~~

... J/ψ production: $\langle \mathcal{O}_1(^3S_1) \rangle$

$\langle \mathcal{O}_8(^3S_1) \rangle$

$\langle \mathcal{O}_8(^1S_0) \rangle$

$\langle \mathcal{O}_8(^3P_0) \rangle$

3 phenomenological

... χ_{cJ} production: $\langle \mathcal{O}_1(^3P_J) \rangle$

$\langle \mathcal{O}_8(^3S_1) \rangle$

1 phenomenological

Heavy Quark Spin Symmetry

approximate symmetry of NRQCD

$\Longrightarrow$ no additional parameters $\langle \mathcal{O}_n \rangle$ required to predict...

... η_c cross section

$\langle \mathcal{O}_8(^3S_1) \rangle$ for J/ψ $\Longrightarrow$ $\langle \mathcal{O}_8(^1S_0) \rangle$ for η_c

$\langle \mathcal{O}_8(^1S_0) \rangle$ $\quad$ $\langle \mathcal{O}_8(^3S_1) \rangle$

$\langle \mathcal{O}_8(^3P_0) \rangle$ $\quad$ $\langle \mathcal{O}_8(^1P_1) \rangle$

... h_c cross section

$\langle \mathcal{O}_8(^3S_1) \rangle$ for χ_{cJ} $\Longrightarrow$ $\langle \mathcal{O}_8(^1S_0) \rangle$ for h

... dependence on quarkonium

$\langle \mathcal{O}_n \rangle$ for unpolarized J/ψ

$\Longrightarrow$ $\langle \mathcal{O}_n \rangle$ for longitudinal J/ψ $m_s =$
transverse

Charmonium production at the Tevatron

mechanisms for J/ψ production

1. J/ψ from B hadrons

$$B \to J/\psi + X \qquad \mathcal{B} \approx 1.1\%$$

2. prompt J/ψ

a. direct J/ψ

b. J/ψ from $\psi(2S)$

$$\psi(2S) \longrightarrow J/\psi + X \qquad \mathcal{B} \approx 54\%$$

c. J/ψ from χ_{cJ}

$$\begin{array}{lll} \chi_{c0} \longrightarrow J/\psi + \gamma & \mathcal{B} \approx & 0.7\% \\ \chi_{c1} \longrightarrow \quad " & \approx & 27\% \\ \chi_{c2} \longrightarrow \quad " & \approx & 14\% \end{array}$$

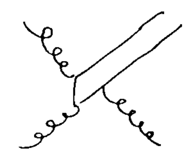
Prompt charmonium at moderate p_T

$$p_T \gtrsim 2m_c$$

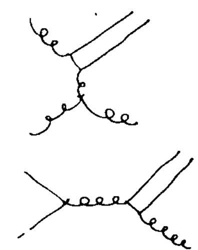
order-α_s^3 parton cross sections

direct J/ψ, ψ'

color singlet: $gg \to g + c\bar{c}_1(^3S_1)$



color octet: $ij \longrightarrow k + c\bar{c}_8(^3S_1)$
$+ c\bar{c}_8(^1S_0)$
$+ c\bar{c}_8(^3P_J)$



direct χ_{cJ}

color singlet: $ij \to k + c\bar{c}_1(^3P_J)$

color octet: $c\bar{c}_8(^3S_1)$

Prompt charmonium production

NRQCD matrix elements

direct J/ψ

$\langle \mathcal{O}_1(^3S_1) \rangle$ — from $J/\psi \to e^+e^-$

$$\left.\begin{array}{l} \langle \mathcal{O}_8(^3S_1) \rangle \\ \langle \mathcal{O}_8(^1S_0) \rangle + \frac{3.5}{m_c^2} \langle \mathcal{O}_8(^3P_0) \rangle \end{array}\right\} \text{ fit to CDF data}$$

direct ψ' same

direct χ_{cJ}

$\langle \mathcal{O}_1(^3P_J) \rangle$ — from $\chi_{c2} \to \gamma\gamma$

$\langle \mathcal{O}_8(^3S_1) \rangle$ — fit to CDF data

Beneke, hep-ph/9703429

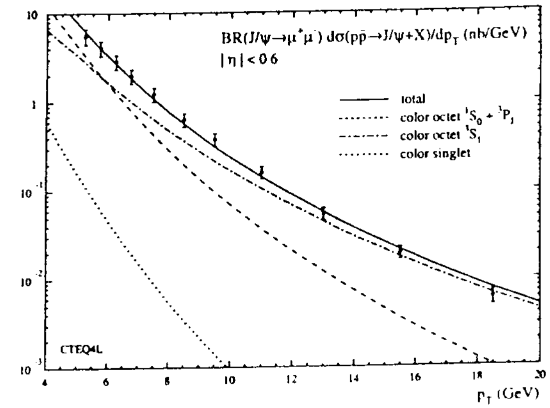


Figure 5: Fit of color octet contributions to direct J/ψ production data from CDF ($\sqrt{s}$ = 1.8 TeV, pseudo-rapidity cut $|\eta| < 0.6$). Theory: CTEQ4L parton distribution functions, the corresponding Λ_4 = 235 MeV, factorization scale $\mu = (p_t^2 + 4m_c^2)^{1/2}$ and m_c = 1.5 GeV. Taken from Ref.[37].

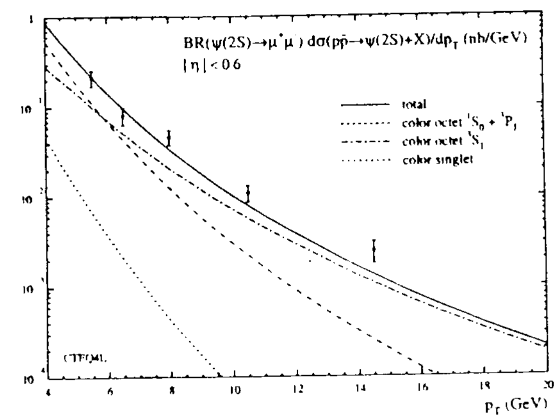


Figure 6: Same as Figure 5 for prompt ψ' production.

From fitting CDF data,

$$\langle \mathcal{O}_8(^3S_1)\rangle \approx 0.01 \text{ GeV}^3$$

$O_1(^3S_1)\rangle$ known from $\psi \to e^+e^-$

Probability density

for point-like $c\bar{c}$ pair

to bind to form J/ψ

$\approx 21 / fm^3$ color-singlet $^3S_1$

$\approx 1.8 / fm^3$ color-octet $^3S_1$

consistent with suppression by v^4

Prompt charmonium at small p_T

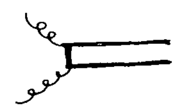
$p_T \ll 2 m_c$

order-α_s^2 parton cross sections ($p_T = 0$)

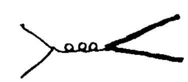
direct J/ψ, ψ'

color-singlet: $\emptyset$

color-octet: $gg \to c\bar{c}_8(^1S_0)$
$c\bar{c}_8(^3P_{0,2})$

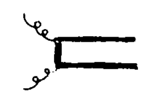


$q\bar{q} \to c\bar{c}_8(^3S_1)$

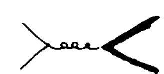


direct χ_{cJ}

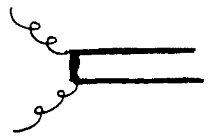
color singlet: $gg \to c\bar{c}_1(^3P_J)$



color octet: $q\bar{q} \to c\bar{c}_8(^3S_1)$

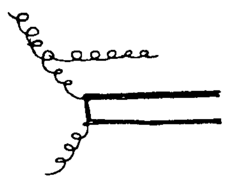


If $\sigma[ij \to c\bar{c}_n] \neq 0$ at order α_s^2,



then at order α_s^3

① $\frac{d\sigma}{dp_T}[ij \to c\bar{c}_n + g] \sim \frac{\alpha_s^3}{p_T}$



from parton splitting $i \to i + g$

② $\sigma[ij \to c\bar{c}_n] = (\text{finite})\,\alpha_s^2 + (-\infty)\,\alpha_s^3$

③ Inclusive cross section

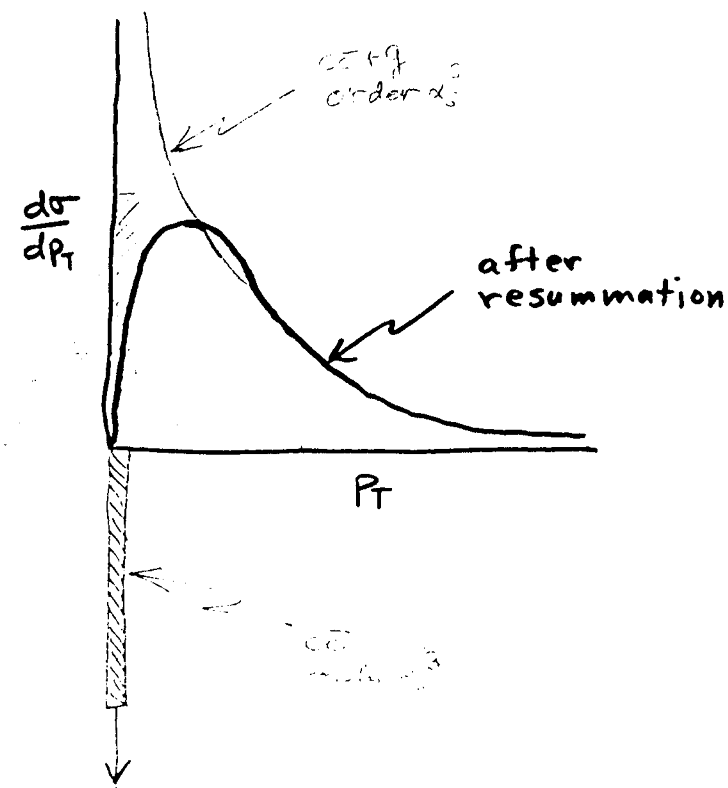
$\sigma[ij \to c\bar{c}_n + X]$

is finite order-by-order in α_s

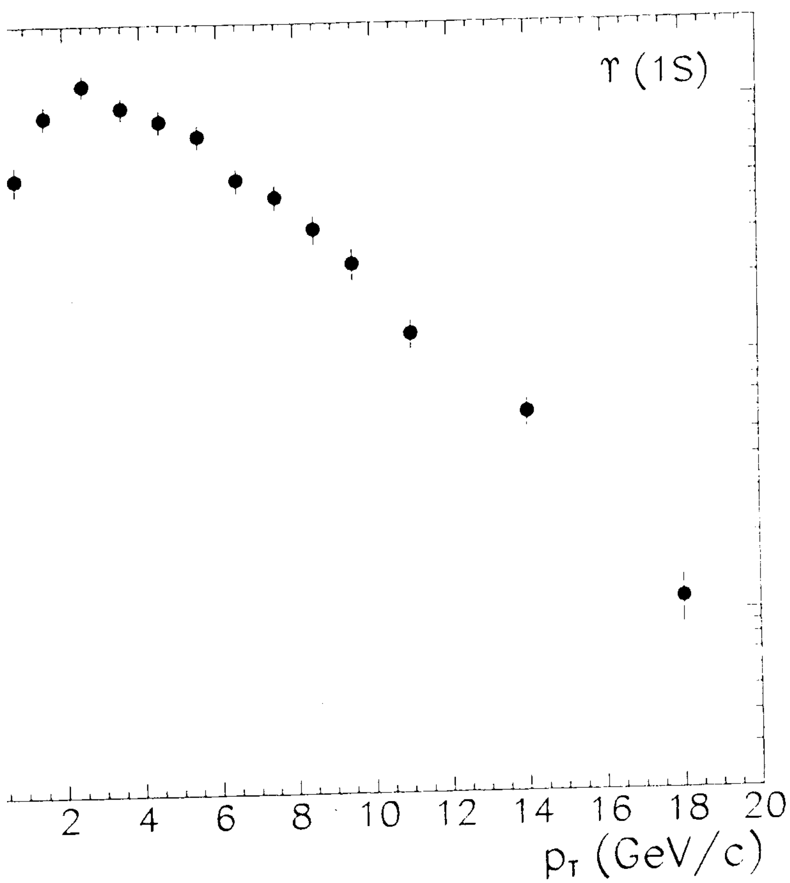
Perturbative p_T distribution

has bad behavior as $p_T \to 0$

Must be resummed!



CDF Preliminary



$B_{\Upsilon \to \mu^+\mu^-} \times \frac{d^2\sigma}{dp_T dy}[p\bar{p} \to \Upsilon + X]\Big|_{y=0}$ vs p_T

Prompt charmonium at large p_T

$p_T \gg 2m_c$

order-α_s^3 parton cross sections

$$\frac{d\sigma}{dp_T^2} \sim \frac{1}{p_T^n}, \quad n = 4, 6, 8$$

direct J/ψ, ψ'

color singlet:	$gg \to g + c\bar{c}_1(^3S_1)$	$\frac{1}{p_T^8}$
color octet:	$ij \to k + c\bar{c}_8(^3S_1)$	$\frac{1}{p_T^4}$
	$ij \to k + c\bar{c}_8(^1S_0) + c\bar{c}_8(^3P_J)$	$\frac{1}{p_T^6}$

direct χ_{cJ}

color-singlet	$ij \to k + c\bar{c}_1(^3P_J)$
color octet	$ij \to k + c\bar{c}_8(^3S_1)$

$$\frac{d\sigma}{dp_T^2} \sim \frac{1}{p_T^4} \Longrightarrow \text{FRAGMENTATION}$$

FACE IT, THE AVERAGE USER WILL NEVER UNDERSTAND FRAGMENTATION

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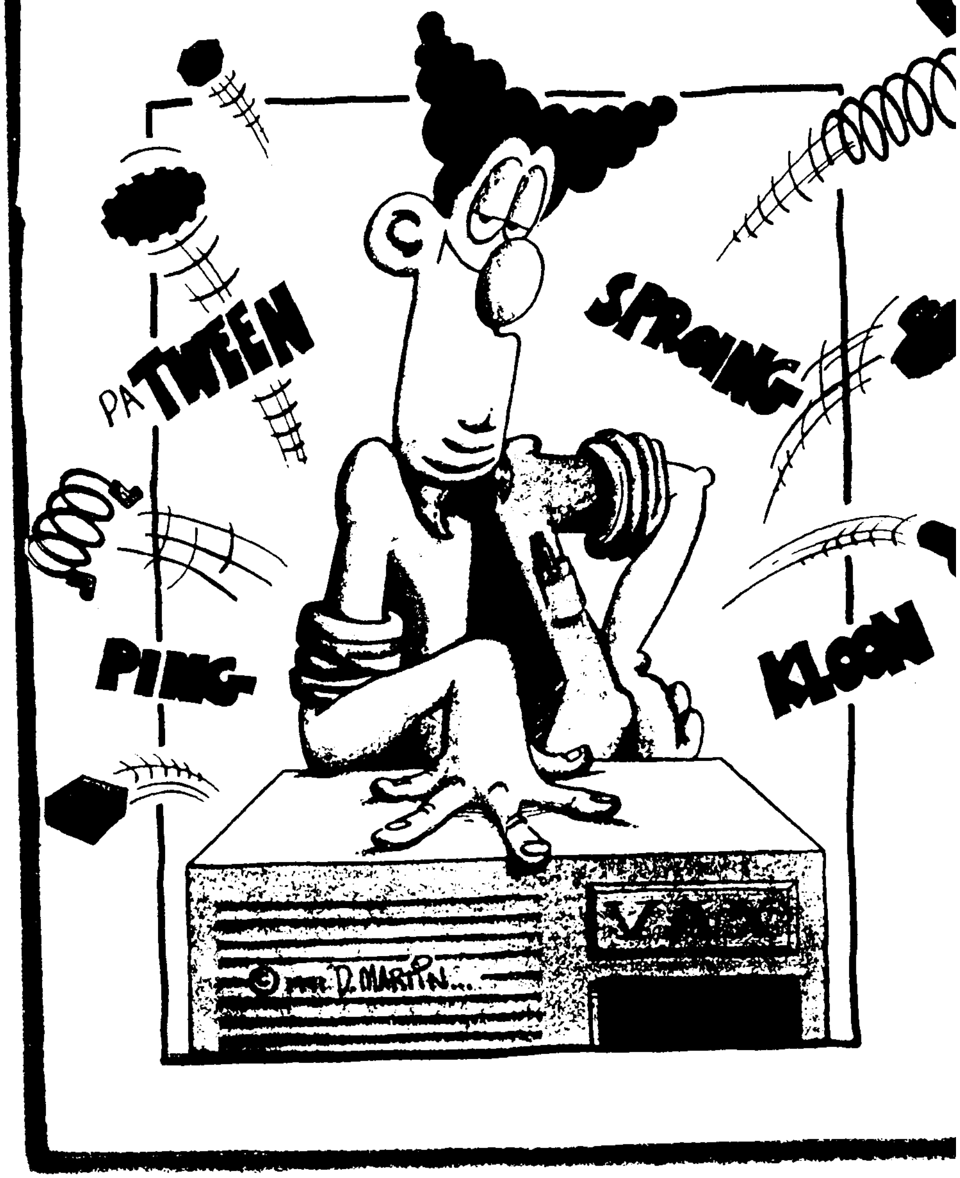


arkonium production at large P_T

dominated by FRAGMENTATION!

$$P_T \gg M \gg Mv,\ Mv^2,\ \Lambda_{QCD}$$

ep 1. Factorization in P_T

separate P_T
from lower scales: $M,\ Mv,\ Mv^2,\ \Lambda_{QCD}$

$$d\sigma[J/\psi + X] = \sum_i d\hat{\sigma}[i+X] \otimes D_{i\to J/\psi}$$

p2 NRQCD factorization

separate M
from lower scales: $Mv,\ Mv^2,\ \Lambda_{QCD}$

$$D_{i\to J/\psi} = \sum_n \hat{D}_{i\to c\bar{c}}\ \langle O_n \rangle$$

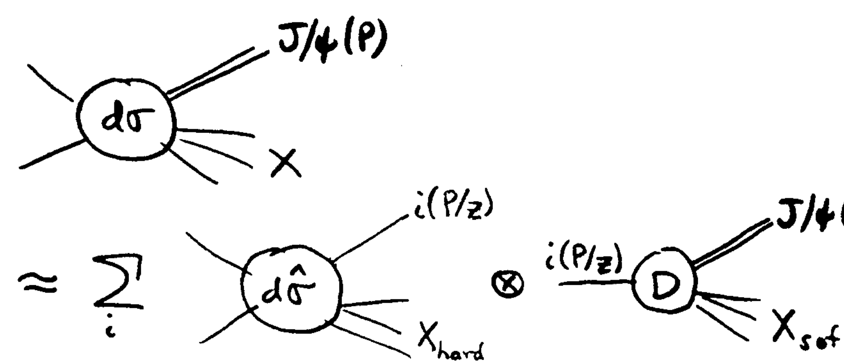
Quarkonium production at large P_T

Step 1 factorization in P_T

$$d\sigma[J/\psi(P) + X]$$
$$\simeq \sum_i \int_0^1 dz\ d\hat{\sigma}[i(\tfrac{1}{z}P) + X]\ D_{i\to J/\psi}(z)$$

$d\hat{\sigma}$ = cross section for producing parton i with transverse momentum $\frac{1}{z}P_T$

$D_{i\to J/\psi}$ = probability for parton i to decay into state that includes J/ψ with momentum fraction z



Quarkonium production at large P_T

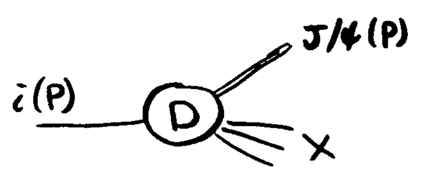
Step 2 NRQCD factorization

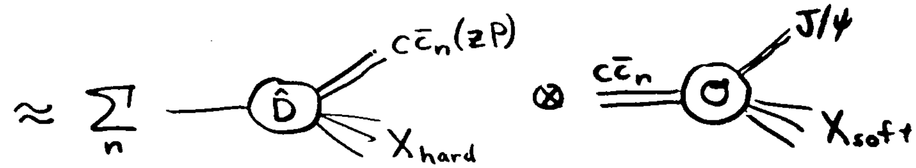
$$D_{i \to J/\psi}(z)$$

$$\simeq \sum_n \hat{D}_{i \to c\bar{c}_n}(z) \langle \mathcal{O}_n \rangle^{J/\psi}$$

$\hat{D}_{i \to c\bar{c}_n}$ = probability for parton i to decay into state that includes $c\bar{c}$ pair in state n with momentum fraction z

$\langle \mathcal{O}_n \rangle$ = NRQCD matrix element





Fragmentation functions for charmonium

NRQCD factorization

$$D_{g \to H}(z) = \sum_n \underbrace{\qquad}_{\text{expansion in}} \langle \mathcal{O}_n \rangle^H$$

color singlet: $g \to c\bar{c}_1(^3S_1)$

color-octet: $g \to c\bar{c}_8(^3S_1)$

$g \to c\bar{c}_8(^1S_0)$
$\to c\bar{c}_8(^3P_J)$

: same

:

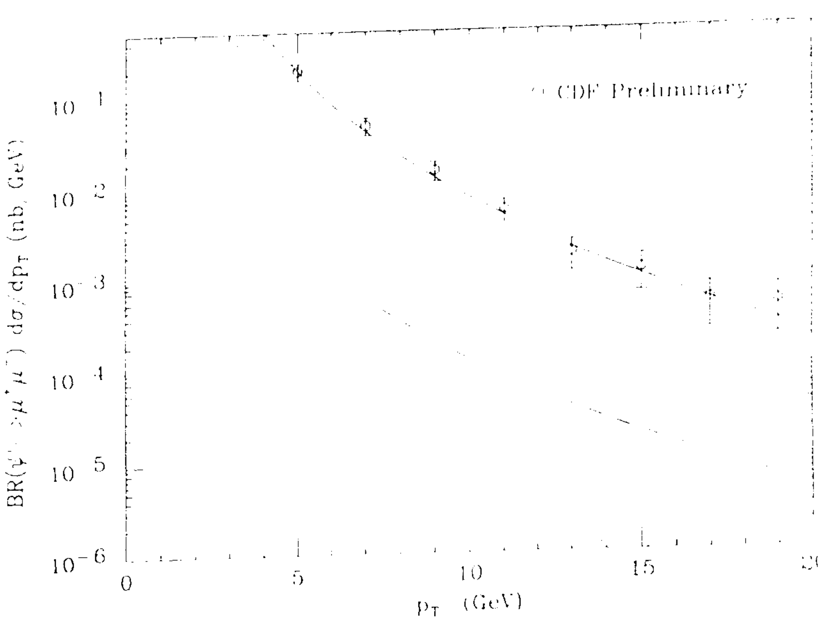
color singlet: $g \to c\bar{c}_1(^3P_J)$

color octet: $g \to c\bar{c}_8(^3S_1)$

CSM
1975-1995
RIP

...... CSM leading order (α_s^3)

— — — CSM fragmentation (α_s^5)



<u>POLARIZATION</u> of Prompt J/ψ, ψ' at the Tevatron

$$p\bar{p} \longrightarrow \psi(\lambda) + X$$

1. At $p_T \gg M_\psi$, σ is dominated by gluon fragmentation

Braaten + Yuan 1993

$$d\sigma(p\bar{p} \to \psi(P) + X) = \int_0^1 dz\, d\sigma\left(p\bar{p} \to g\left(\tfrac{P}{z}\right) + X\right) D_{g\to\psi}(z)$$

2. Gluon fragmentation may be dominated by color-octet matrix element

Braaten + Fleming 1995

$$D_{g\to\psi}(z) = \frac{\pi \alpha_s}{24 m_c^3}\, \delta(1-z) \left\langle O_8(^3S_1)\right\rangle$$

3. At leading order in α_s,

gluon fragmentation produces ψ's that are 100% transversely polarize.

Cho + Wise 1995

4. Beyond leading order in α_s

Beneke + Rothstein 199

polarization from gluon fragmentation

$> 90\%$

5. Beyond fragmentation

Beneke + Kramer 1996
Leibovich 1996

dominant corrections to polarization come from $1/p_T^2$ corrections

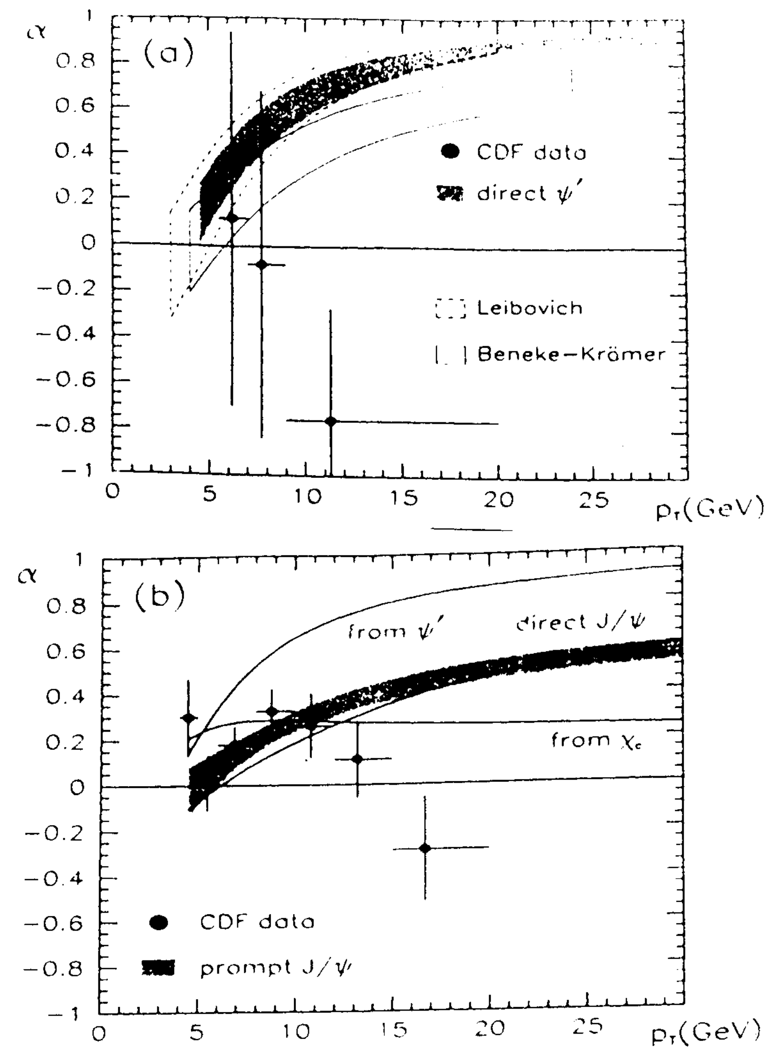


FIG. 1. Polarization variable α vs. p_T for (a) direct ψ' and (b) prompt J/ψ compared to CDF data.

Production of Charmonium at large P_T

At sufficiently large P_T,

J/ψ has helicity ± 1

1. PQCD Factorization

$$\sigma(J/\psi) = \hat{\sigma}(g) \otimes D_{g\to\psi}$$

2. NRQCD Factorization

$$D_{g\to\psi} = \sum_n \hat{D}_{g\to c\bar{c}_n} \langle \mathcal{O}_n \rangle$$

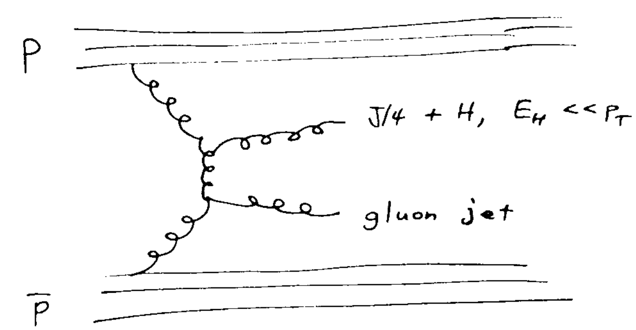
3. $c\bar{c}_8({}^3S_1)$ term dominates

$\Rightarrow$ J/ψ has helicity ± 1 !

Step 1 PQCD Factorization

Separate momentum scale from smaller scales $m_c, \ldots, \Lambda_{QCD}$

σ is dominated by fragmentation of higher P_T gluon



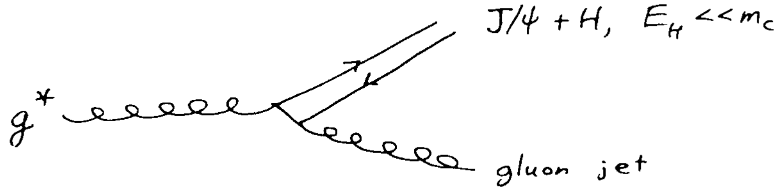
$$d\sigma(J/\psi(P)) \approx \int_0^1 dz \qquad D_{g\to\psi}(z)$$

power series in $\alpha_s(P_T)$

Step 2. NRQCD Factorization

Separate momentum scale m_c

from lower scales $m_c v$, $m_c v^2$, Λ_{QCD}



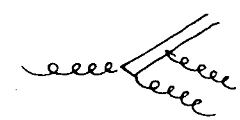
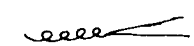
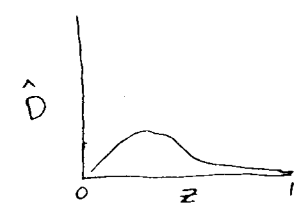
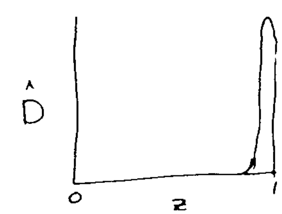
$$D_{g\to\psi}(z)$$

$$= \sum_n \hat{D}_{g\to c\bar{c}_n}(z) \langle O_n^{J/\psi} \rangle$$

power series in $\alpha_s(m_c)$

Claim: $c\bar{c}_8(^3S_1)$ term dominates

$c\bar{c}_8(^3S_1)$ term dominates !

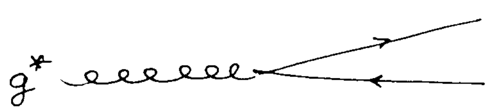
	$c\bar{c}_1(^3S_1)$	$c\bar{c}_8(^3S_1)$
subprocess	$g^* \to c\bar{c} + gg$	$g^* \to c\bar{c}$
		
short-distance factor	$\hat{D} \sim \alpha_s^3$	$\hat{D} \sim \alpha_s$
long-distance factor	$\langle O \rangle \sim v^3$	$\langle O \rangle \sim v^4 \times v^3$
z distribution		
$\sim$	$\sim \langle z \rangle^4 \alpha_s^3(m_c)\, v^3$	$\sim \alpha_s(m_c)\, v^7$

If $D_{g \to \psi}$ is dominated by $c\bar{c}_8({}^3S_1)$,

then **helicity** of J/ψ at large p_T

will be **transverse**!

Cho + Wise 1995



An almost on-shell gluon has helicity ± 1

$\Longrightarrow$ $c\bar{c}$ has helicity ± 1

$\langle O_8({}^3S_1)^{J/\psi} \rangle$ is dominated by momenta $\lesssim mv$

$\Longrightarrow$ spin-flip interactions suppressed by powers of v^2

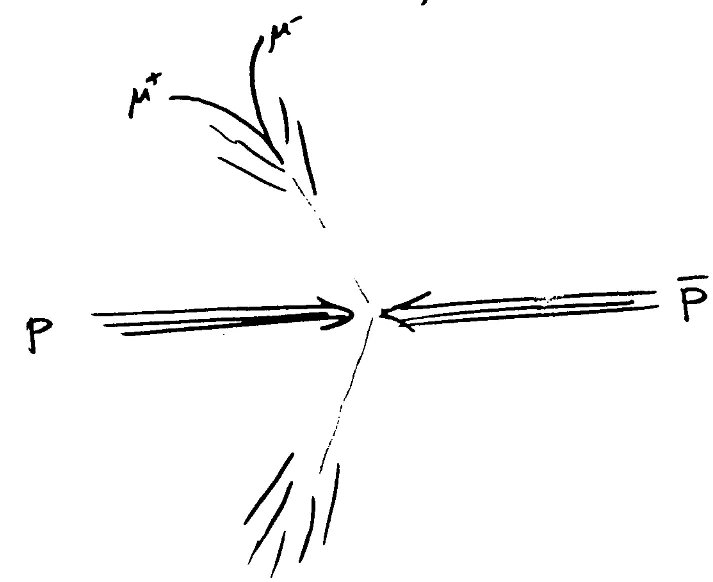
$\Longrightarrow$ J/ψ has helicity ± 1

Quarkonium

for flavor tagging of jets?

Jets with large E_T ($E_T \gtrsim 100$ GeV)

and "soft collinear" $\mu^+\mu^-$ ($M_{\mu^+\mu^-} \sim 3$ GeV)



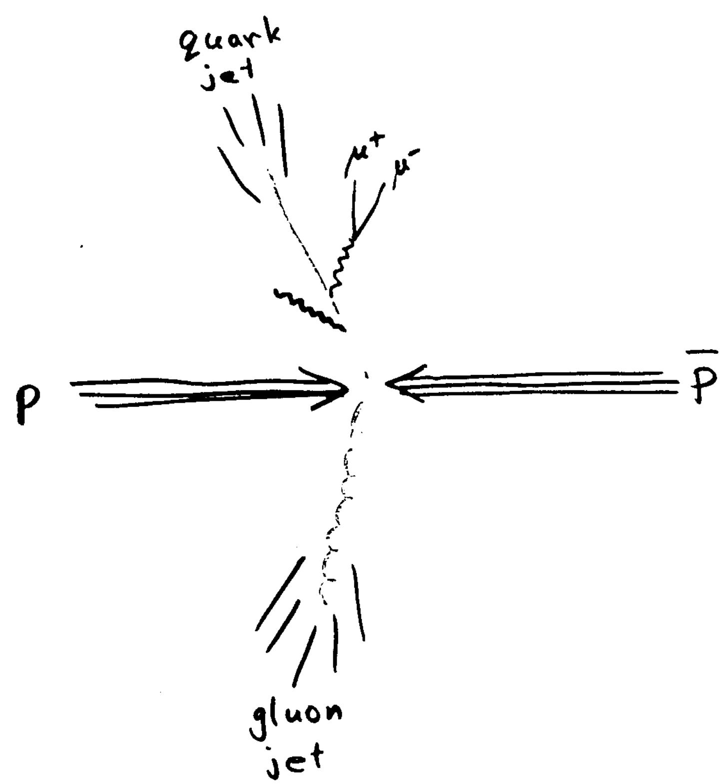
$M_{\mu^+\mu^-} = M_{J/\psi}$ $\Longrightarrow$ bias toward

$M_{\mu^+\mu^-} \neq M_{J/\psi}$ $\Longrightarrow$ bias toward

Soft collinear" dileptons

follow same radiation pattern

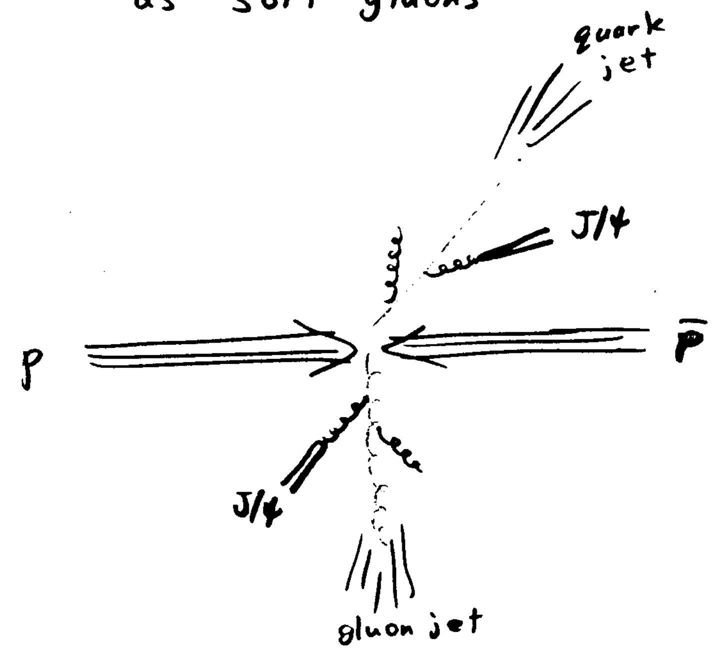
as soft photons



"Soft collinear" J/ψ

follows same radiation pattern

as soft gluons



Fragmentation probability:

$$\int_0^1 dz\, D_{g \to J/\psi}(z) = \text{[illegible]}$$

$$\approx 10^{-4}$$

onclusions

?QCD Factorization

systematic framework for describing

annihilation decay rates

inclusive cross sections for quarkonium

r quarkonium production ...

. quantitative analysis is premature

need NLO calculations,

soft gluon resummation, ...

dramatic qualitative predictions

- polarization of J/ψ at
- radiation pattern of "soft collinear" J/ψ