Diffraction

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CTEQ Summer School 2000

1 Soft diffraction
- Optical model for high-energy hadron scattering
- Light-cone variables, rapidity
- Final state in hadronic collisions
- Pomeron
- Space-time picture
- Diffractive excitation and rapidity gaps

2 Diffraction and hard scattering
- Diffractive DIS (and other processes)
- Ingelman-Schlein model
- Data: fits and predictions
- Theory
- Mechanisms
- Factorization breaking
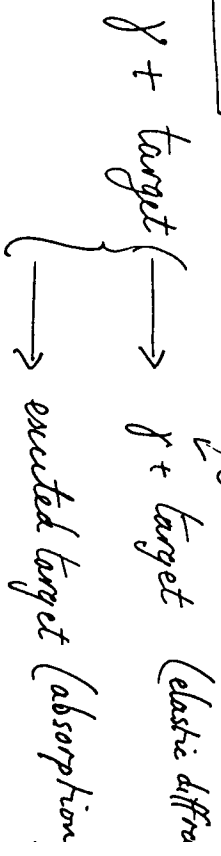- Exclusive DIS and skewed parton distributions

Wave fronts impinging on target — absorbing or.

Diffraction pattern when $\lambda \ll$ size of target, small angles. Black disk $\Longrightarrow$
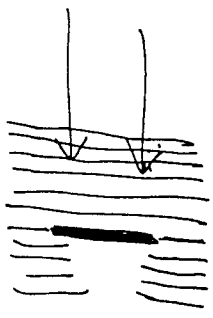
Processes

γ + target $\begin{cases} \longrightarrow \\ \longrightarrow \end{cases}$

isolated, forward.

γ + target (elastic diffr

excited target (absorption

(Zhou & Yang) PR **170**, 1591 (1968)

Wave function for CM of hadron relative to other hadron
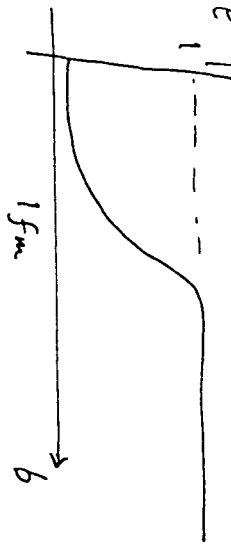
Target: Lorentz contracted hadron

Just after disk $\qquad e^{ipz}\, e^{-\Omega(b;s)}$

Ω is opaqueness: $\begin{cases} 0 & \text{for invisible target} \\ 1+\text{phase} & \text{for transparent targ.} \\ \infty & \text{for total absorption} \end{cases}$
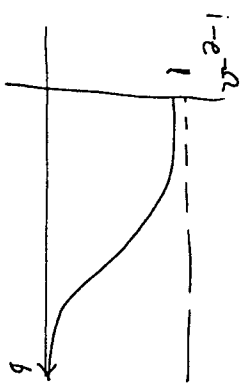
$\therefore$ Expect

$e^{ipz} + const\; \dfrac{e^{ipr}}{r}\, M(q_T)$

Scattering amplitude

$$M(q_T) = is \int d^2b\; e^{-iq_T\cdot b}\left(1 - e^{-\Omega(b;s)}\right)$$
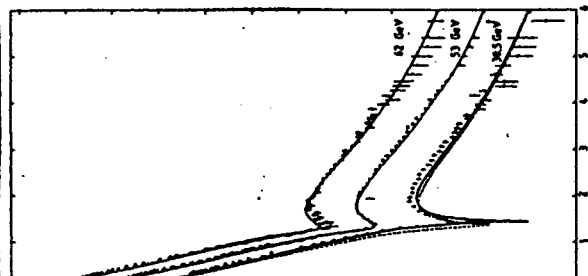
Typical profile

$$\frac{d\sigma}{dt} = \frac{|M|^2}{16\pi s^2}$$

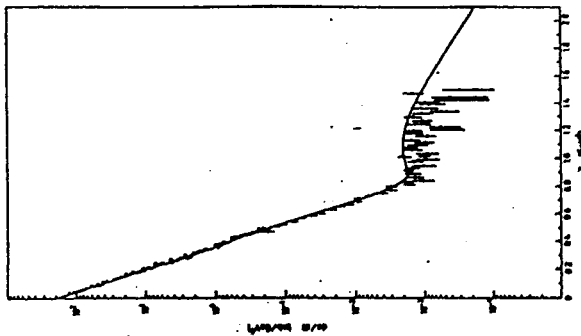$$\sigma_{tot} = \frac{1}{s}\, \text{Im}\, M(q_T=0)$$
$$= \int d^2b\; \text{Re}\left(1 - e^{-\Omega}\right) \qquad \text{(optical theorem)}$$
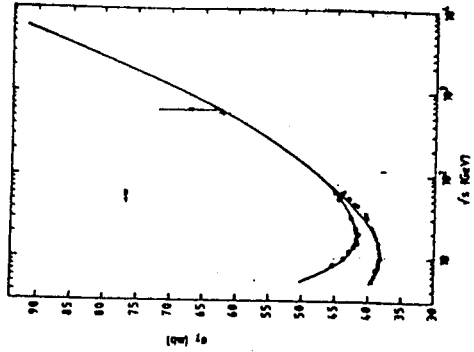
Data $\Rightarrow$ 1. σ_{tot} varies slowly w/s: "$s^{0.08}$"

2. $\frac{d\sigma}{dt}$ falls rapidly w/t: e^{Bt}, $B=17\,GeV^{-2}$ at $\sqrt{s}=550\,GeV$.

3. $\frac{d\sigma}{dt}$ narrows as s increases
— "proton gets bigger", Ω increases

Exercises on sizes

1. rms(b) of $1-e^{-\Omega}$ is $\sqrt{2\frac{d}{dt}\ln\frac{d\sigma}{dt}}$ = $\sqrt{2B}$

$\approx 1.2\,fm$ at $\sqrt{s}=550\,GeV$

2. Black disk of radius R $\Rightarrow$ R = 0.8 fm $\sqrt{\dfrac{1\,GeV^2}{-t_{1^{st}min}}}$

(uses $\theta_{1^{st}min} = 1.2\,\lambda/d$)

R = 0.9 fm for $\sqrt{s}=550\,GeV$

rms(b) = $R/\sqrt{2}$ (for black disk).

Proton is fairly black

$\therefore \sigma_{tot} = 60\,mb = \pi \times (1.4\,fm)^2$.

$\frac{d\sigma}{dt}(p\bar{p})$

$(p\bar{p})$

Contrast s-channel graph, off resonance

$\Longrightarrow$

$$\frac{P_0 + P_2(\cos\theta)}{1+\cos^2\theta} \quad \text{for } \gamma^*, z_c$$
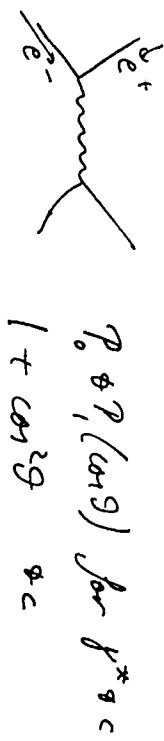
1. Roughly constant in angle
2. Large P_T typical
3. Small J.
4. $s \propto 1/s$.

<u>Hadron-hadron</u>

1. Typically many particles in final state, uniform in rapidity
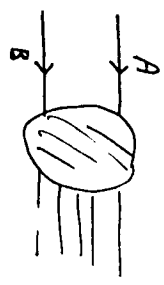2. Small transverse momentum
3. Approx. constant cross sections
4. ⟨Impact parameter⟩ ~ constant
   ⟨J⟩ ∝ $\sqrt{s}$

Define $(V^+, V^-, V_T) = \left(\dfrac{V^0+V^z}{\sqrt{2}}, \dfrac{V^0-V^z}{\sqrt{2}}, V_T\right)$

1. z-boost: $V^+ \longrightarrow V^+ e^y$
   $V^- \longrightarrow V^- e^{-y}$

2. $V \cdot W = V^+ W^- + V^- W^+ - V_T \cdot W_T$.

Particle momentum $k^\mu = \left(\dfrac{E_T}{\sqrt{2}} e^y, \dfrac{E_T}{\sqrt{2}} e^{-y}, k_T\right)$,
where $E_T = \sqrt{k_T^2 + m^2}$
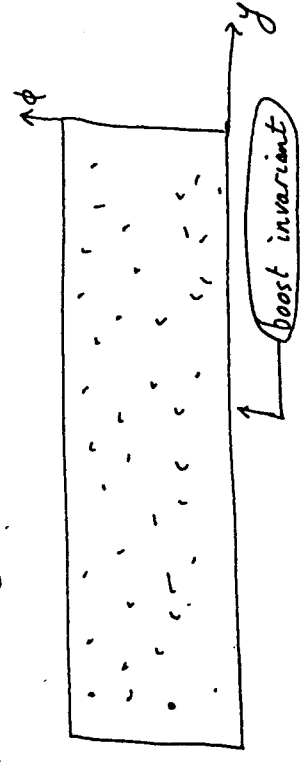& $y \overset{def}{=} \dfrac{1}{2} \ln \dfrac{k^+}{k^-}$: rapidity. Boost: $y \longrightarrow y$

Usefulness: — simple tfm under boosts
— easy to find large & small terms in $V \cdot W$
— approx. uniform particle distribution in
  H-H w.r.t. beam axis
  jet w.r.t. jet axis

Neglect m: $y \approx -\ln \tan \dfrac{\theta}{2} = \eta$ (pseudo-rapidity)

Normal inelastic

boost invariant

Detector: θ not y or η
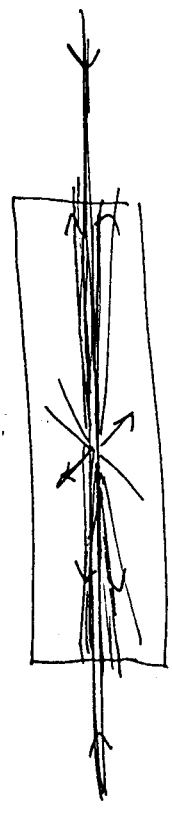
About 2 hadrons/unit of y, typical $p_T \simeq 0.35$ GeV.

CM
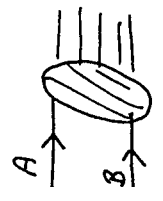
$$p_A^r = \left(\sqrt{\tfrac{s}{2}}, \, \frac{m^2}{\sqrt{2s}}, \, 0_T\right)$$

$$y_A = \ln\sqrt{\frac{s}{m}} = 7.6 \text{ at Fermilab}$$

$$p_B^r = \left(\frac{m^2}{\sqrt{2s}}, \, \sqrt{\tfrac{s}{2}}, \, 0_T\right)$$

$$y_B = -7.6 \quad \& \quad \Delta y = 15.2$$

at small angle.

$$p_A' \simeq \left(\sqrt{\tfrac{s}{2}}, \, \frac{m^2+p_T^2}{\sqrt{2s}}, \, p_T\right)$$

$$y_A' = y_A - \ln\frac{\sqrt{m^2+p_T^2}}{m} = y_A - (\text{less than 1 unit})$$

y or η

Spin of exchanged system
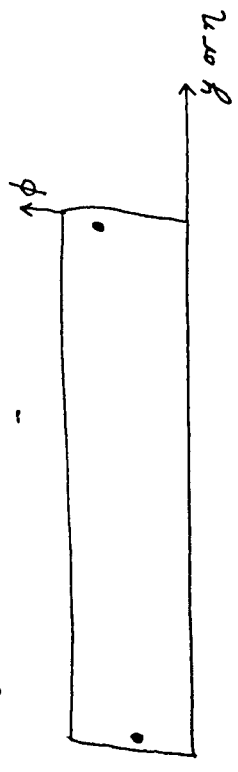
Feynman graph, large s, small angle: $|t| \ll s$

$$= S^J f_n(t) \implies \frac{d\sigma}{dt} = S^{2(J-1)} f_n(t)$$

$$J^\pi = \left(O\left(\frac{r^2}{\sqrt{s}}\right), \; O\left(\frac{r^1}{\sqrt{s}}\right), \; P_T \right)$$
$$\quad\quad + \quad\quad\quad\quad - \quad\quad\quad T$$

analytically continued $Y_m(\cos\theta)$

Multiple exchange
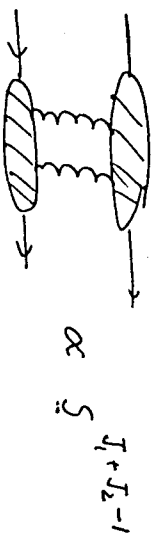
$$\propto S^{J_1+J_2-1}$$

$$J_1 = J_2 = 1 \;\&\; c \implies \text{const cross sect. w/}$$
enhancements
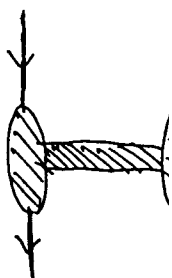
Regge theory:

1. Associate power-law in s with exchanged σ

$$S^{\alpha(t)} f(t)$$
$$\uparrow \text{ effective spin}$$

2. Treat exchanged object as (analytically continued) particle.

3. σ rises slowly $\iff \alpha = 1 + $ small
   Exchanged object is called Pomero

4. But it's not really a particle ...
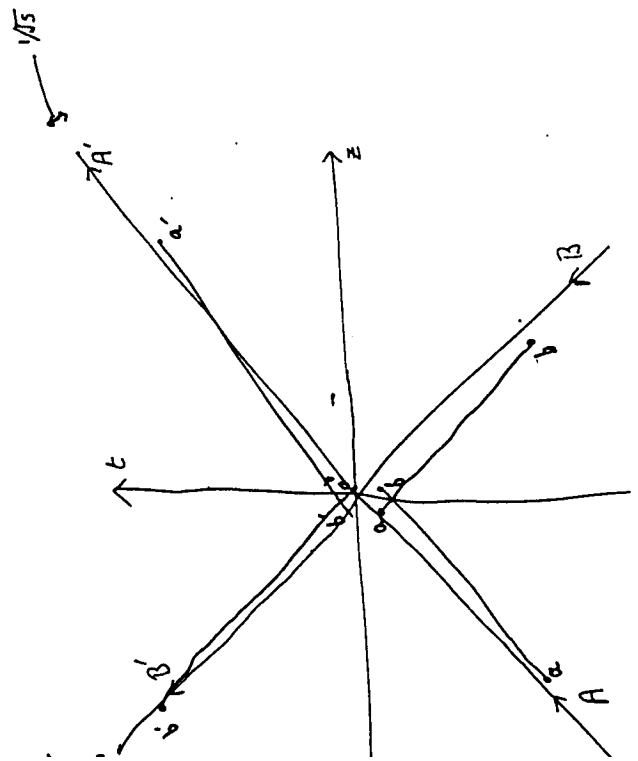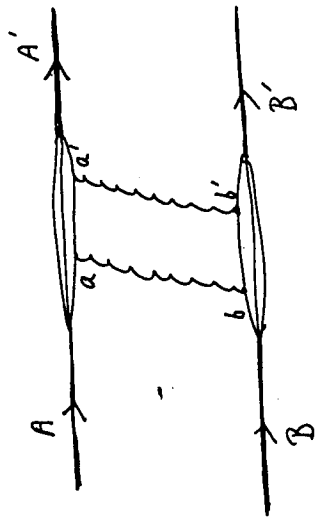
Basics of final-state production

1 or 2 particles/unit y

Fuzzy: non-perturbative physics, not Feynman graphs
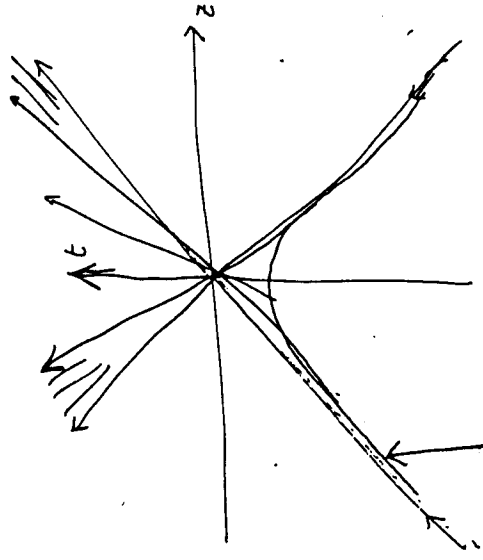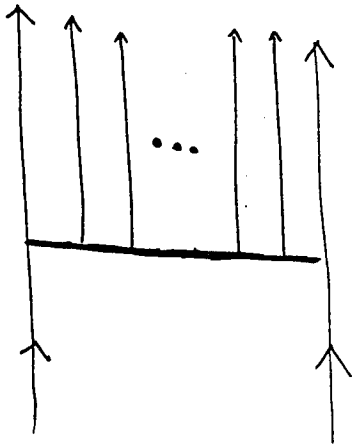
Exchanges on SPACE-LIKE fuzzy hyperbola $c^2 t^2 - z^2 \sim 1\,fm^2$.

t z

ce-time

alize from

A' a' A

B' b' B

A' a' $\frac{1}{\sqrt{s}}$ s

t z

B' b'

A a B b
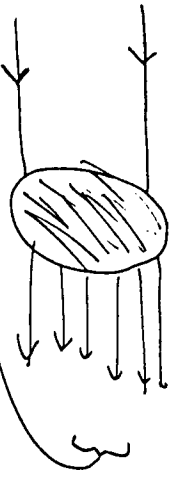
Locally a boost invariant system

Big theory issue — how to formulate this in terms of QM states (while obeying Bell's thm. & c)?

Similar issues in $e^+e^- \longrightarrow$ jets & c.

Cause
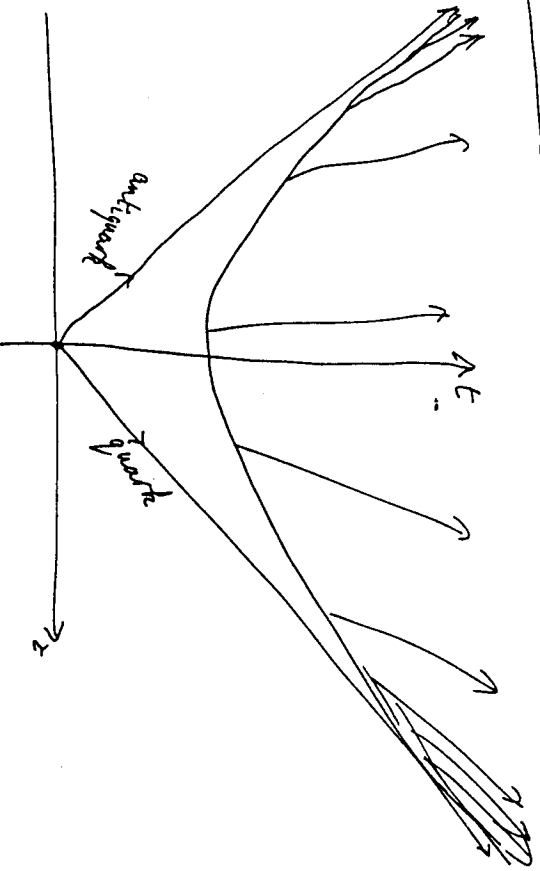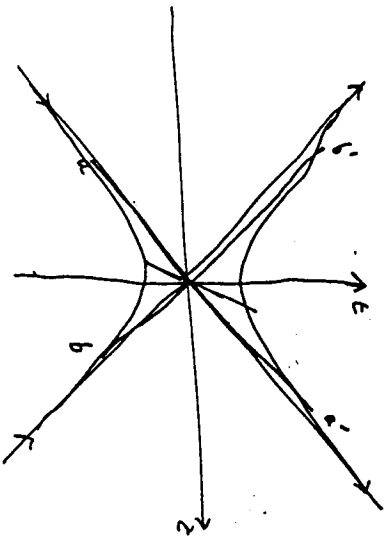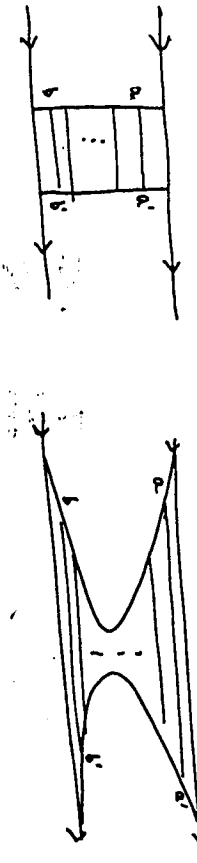
Why?

1. fastest right movers at top
   — time scale $\frac{\sqrt{s}}{m} \cdot \frac{1}{m}$

2. fastest left movers at bottom

3. Change time ordering by boost
   — hence t-channel lines space-like.

Hadronic diffraction

Optical diffraction : $\begin{cases} \text{elastic} \\ \text{absorption} \end{cases}$
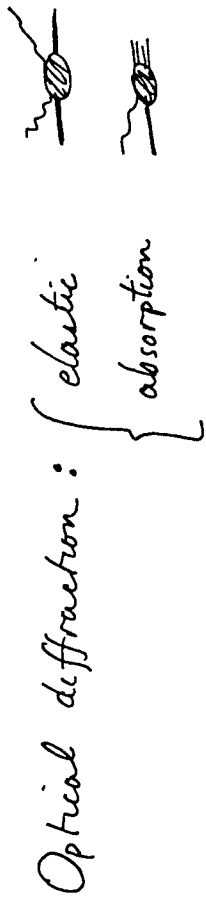
Hadronic diffraction — also the possibility of "diffractive excitation"

← small energy loss

← rapidity gap
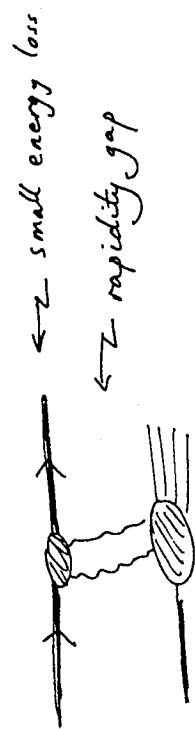
can also do this in ep collisions
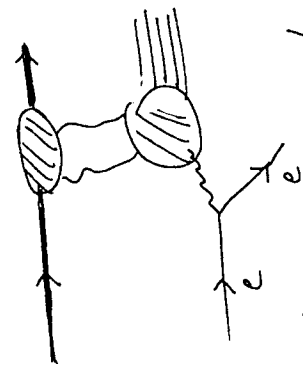
MANY variations on theme.

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not particle exchange?

$p(1-x)$

px

energy denominator
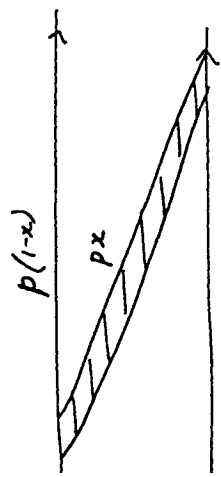
$\left(\, + \frac{m^2}{2p}\right) - \left(p(1-x) + \frac{m^2+p_T^2}{2p(1-x)}\right) - \left(px + \frac{m^2+p_T^2}{2px}\right)$

$= -\frac{p_T^2 + m^2(1-x+x^2)}{2px(1-x)}$

$\sim \frac{px}{m^2}$ if $x \ll 1$ & $p_T \sim m$

c.w. p/m^2.

Currently:

"diffraction" is used to label hadronic scattering with large rapidity gaps. Typically diffractive excitation is investigated.

This area least resembles optical diffraction!

153 GeV - elec

ETA PHI

LCAL-transverse energy
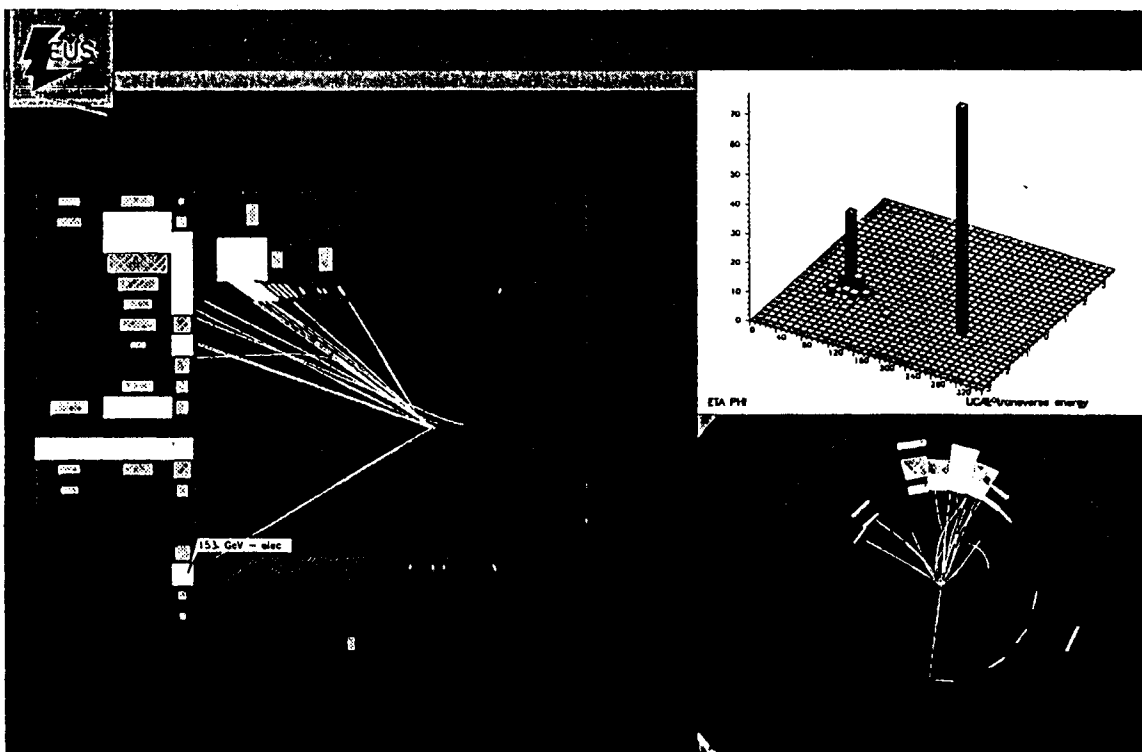
Kinematics of diffraction
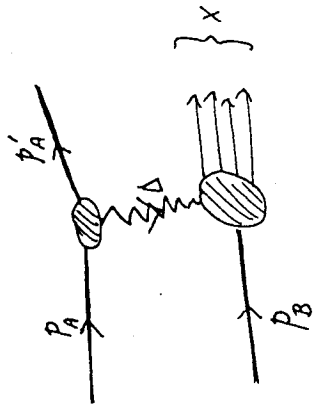
$$P_A^r = \left(P_A^+ = \sqrt{\tfrac{s}{2}}, \; \frac{m^2}{2P_A^+}, \; 0_T\right)$$

$$P_B^r = \left(\frac{m^2}{2P_B^-}, \; P_B^- = \sqrt{\tfrac{s}{2}}, \; 0_T\right)$$

$$\Delta^r = \left(x_P P_A^+, \; \ldots, \; -\vec{P}_{AT}\right)$$

Diffraction: $x_P \ll 1$;

$$x_P = \Delta^+/P_A^+ = \frac{m_x^2}{s} = \sum_{i \in x} \frac{E_{Ti}\, e^{y_i}}{P_A^+ \sqrt{2}} \simeq \sum_{\text{highest } \sigma E_T} \frac{E_{Ti}\, e^{y_i}}{2E}$$

$$P_A^{'r} = \left(P_A^+(1-x_P), \; \frac{m^2 + P_{AT}'}{2(1-x_P)P_A^+}, \; \vec{P}_{AT}'\right)$$

$$t \simeq -P_{AT}'^2$$
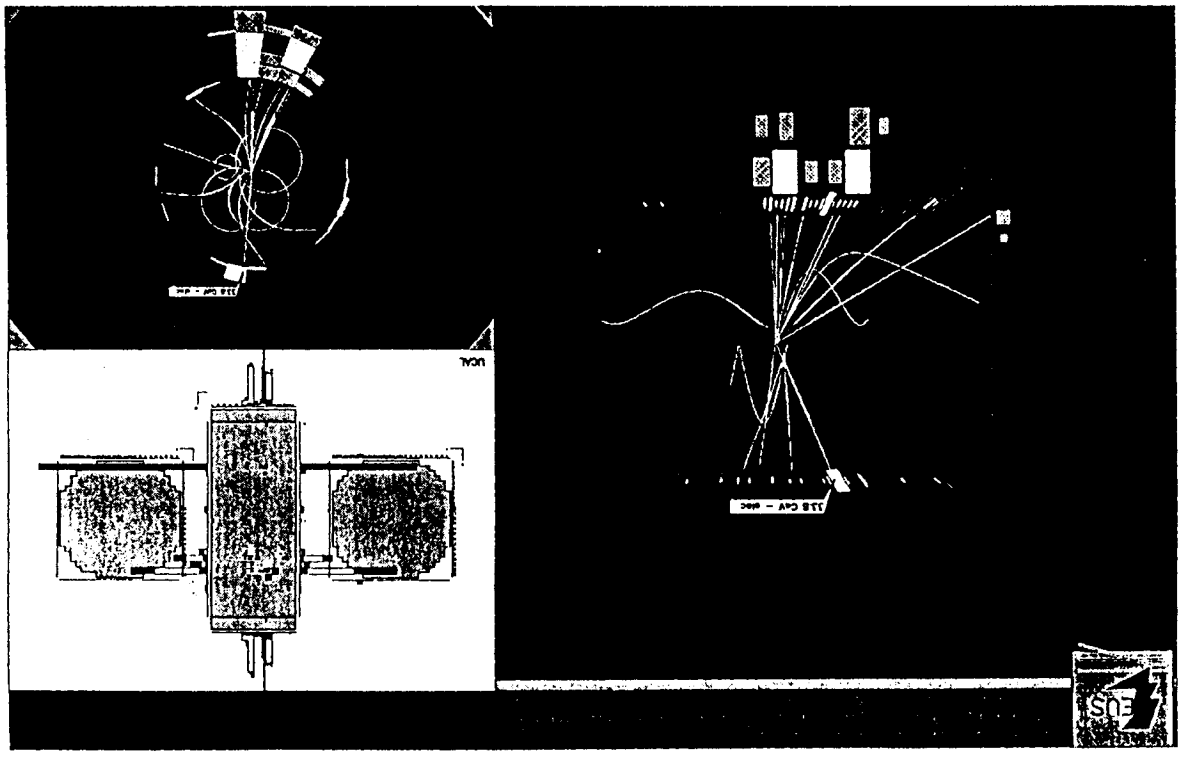
Triple-Pomeron formula

$$A + B \longrightarrow A' + X$$

$\rightarrow M_x^2 = x_{\mathbb{P}} s$

Lowest order Regge theory — treat $\{\}$ as particle.

$x_{\mathbb{P}} \ll 1, \quad M_x \gg 1\,GeV$

$$x_{\mathbb{P}} \frac{d\sigma}{dx_{\mathbb{P}} \, dt} = \frac{1}{16\pi} |\beta(t)|^2 \, \beta(0) \, g_{\mathbb{PPP}}(t) \cdot$$

$$\cdot \left(M_x^2\right)^{\alpha_{\mathbb{P}}(0)-1} \left(\frac{1}{x_{\mathbb{P}}}\right)^{2[\alpha_{\mathbb{P}}(t)-1]}$$

Predictions 1. Power laws in M_x & $x_{\mathbb{P}}$

  2. t-slope lower than elastic

  3. $\sigma_{SDE} > \sigma_{tot}$ at high energy

But

$\longrightarrow \quad \cdots \longrightarrow OUT$

$$x_{\mathbb{P}} \approx \frac{\langle E_{T_i} \rangle}{2E} \int_{y_A}^{y_{max}} dy \frac{dN}{dy} e^y$$

$$= e^{y_{max}} \frac{\langle E_{T_i} \rangle}{2E} \frac{dN}{dy}$$

$\underset{Typical}{=}$ rap. 0.9

$$\ln \frac{1}{x_{\mathbb{P}}} = \underbrace{y_A - y_{max} + \ln \frac{m}{\langle E_{T_i} \rangle} - \ln \frac{dN}{dy}}_{constant}$$

$$\underbrace{\qquad\qquad\qquad}_{y_{max}} \qquad \underbrace{\qquad}_{y_A}$$

$\to \eta$ or

$\eta_{max} \uparrow$

Inclusive-Diff Calorimeter Tower η_{max} vs $\ln(\xi)$

CDF Preliminary

Detected β_{max} (west)

$\ln x_{\mathbb{P}} \longrightarrow$
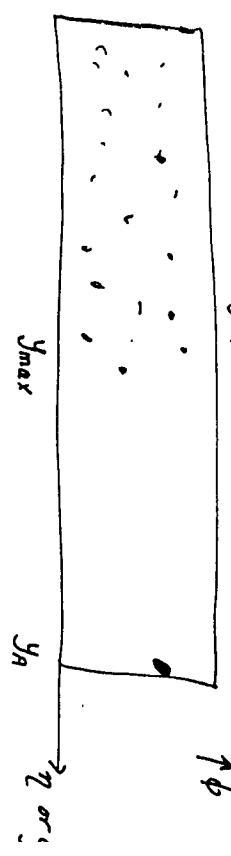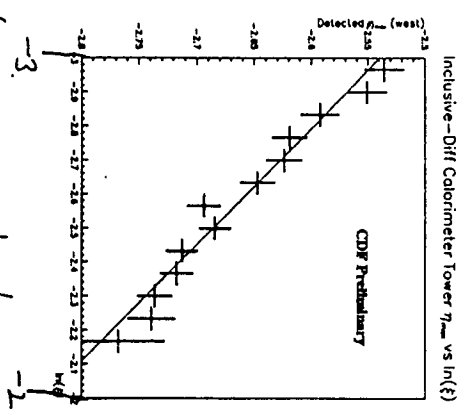
Changes if there are high P_T-jets

Diffraction + hard scattering

Since QCD: $\sqrt{s}$ has increased.

Can have diffraction — e.g. $\dfrac{M_x^2}{s} = 10^{-2}, 10^{-3}$

& have M_x^2 large enough to have hard scattering → calculable

Processes:

1. Classical diffractive excitation plus high P_T — eg. DIS, jets.

2. Elastic electroproduction $\gamma^* p \to \ell\, p\, q\, c.$

3. Nuclear targets.

Gives microscope onto strong interaction scattering

ding proton in diffractive DIS:

ZEUS 1994
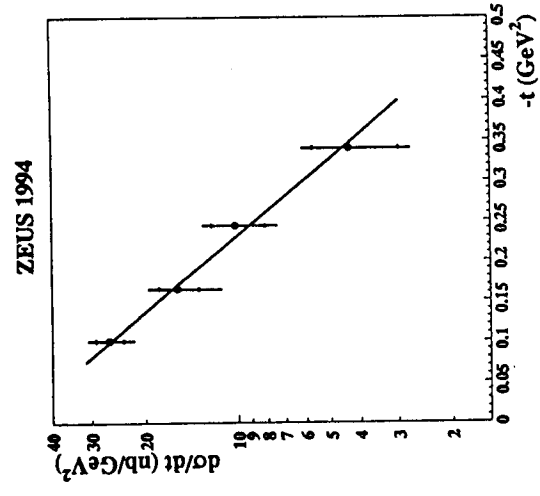
$$\frac{d\sigma}{d|t|} \propto e^{-b|t|} \quad \text{with} \quad b = 7.2 \pm 1.1\,{}^{+0.7}_{-0.9}\ \text{GeV}^2$$

at $\sqrt{s} = 550\ \text{GeV}$: $b \simeq 14\ \text{GeV}^2$

ep $\longrightarrow$ e X p

Treat Pomeron as particle & apply usual pQCD methods to DIS ($\Phi c.$) on Pomeron too. Is it QCD? Yes — later. But...

Lowest order in $\alpha_s(Q)$:

"current quark" jet

"Pomeron beam remnant"
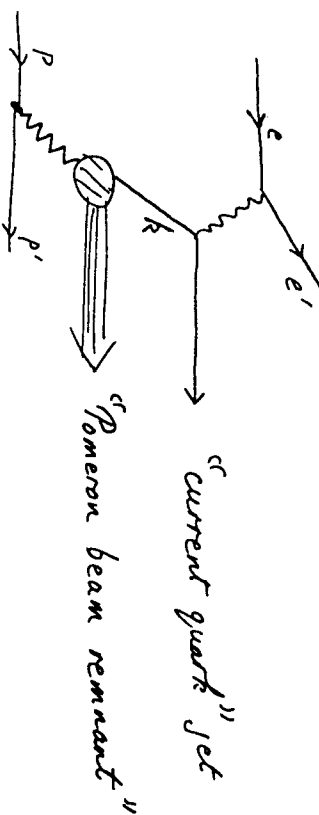
$$k^\mu = (\xi x_P p^+, \text{ small}, \text{ small})$$
$$+ \; q^\mu = (-x_{bj} p^+, \frac{Q^2}{2 x_{bj} p^+}, 0_T)$$
$$\overline{(\text{current jet})^\mu = (\text{small}, \frac{Q^2}{2 x_{bj} p^+}, \text{small})}$$

Hence $\xi \approx \dfrac{x_{bj}}{x_P} = \beta$.

Variables Q^2, x_{bj}; ξ, x_P.

$$p^\mu = \left(p^+, \frac{m^2}{2p^+}, 0_T\right)$$
$$q^\mu = \left(-x_P p^+, \frac{Q^2}{2x_P p^+}, 0_T\right)$$
$$-p'^\mu = \left(x_q p^+, O(m^2/p^+), O(m)\right)$$

$$x = x_{bj} \stackrel{\text{def}}{=} \frac{Q^2}{2 p \cdot q} = \frac{-q^+}{p^+} = \cdots$$

$$x_P = \frac{p^+ - p'^+}{p^+} = \cdots$$

Define $\beta = \dfrac{x_{bj}}{x_P} = $ Bjorken x w.r.t. Pomeron

$(0 \le \beta \le 1,$ instead of $0 \le x_{bj} \le x_P)$

Inclusive diffraction — data

HERA (H1 & ZEUS)

- DIS (rap.gap & "Roman pot")
- dijets in DIS & photoproduction
- Charm

Tevatron (CDF & DØ)

- dijets
- W, Z
- bottom

- dijets in double Pomeron exchange

$$F(\beta, Q^2; x_{I\!\!P}, t) = \sum_i e_i^2 \beta f_{i/I\!\!P}(\beta, Q^2) \cdot$$

flux factor $= |\beta_{I\!\!P I\!\!P}(t)|^2 \left(\frac{1}{x_{I\!\!P}}\right)^{2[\alpha_{I\!\!P}(t)-1]}$

...ization hypotheses:

Regge ⟸ $x_{I\!\!P}$-dep; t-dep.

Hard scattering ⟹ other processes, Q-dep.

these correct?

ey follow from QCD?

- predictions, using fits?

"Structure function" & "parton density"
are different concepts.

HERA)

→ e' at high p_T
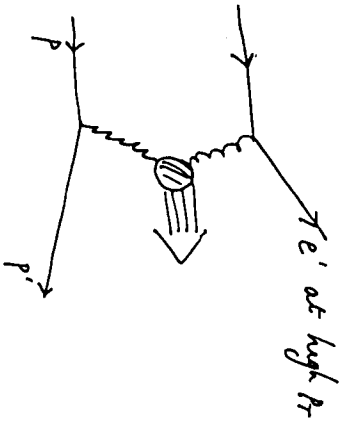
Outgoing proton $p_L = 820 \text{ GeV} (1-x_p)$
$\simeq 820 \text{ GeV}$

$\langle p_T \rangle \sim 0.4 \text{ GeV}$

$\therefore \theta \sim 5 \times 10^{-4}$ radian

ZEUS : LPS (leading proton spectrometer)
23 — 90m downstream
'Roman pot' in beam pipe
Small acceptance

ZEUS 1994

Events

proton   ZEUS PRELIMINARY 1995

neutron   ZEUS PRELIMINARY 1995-19

31

H1 1994 Data

$x_P F_2^{D(3)}$

x_P
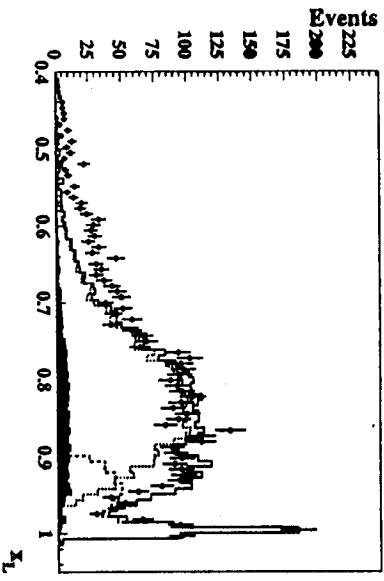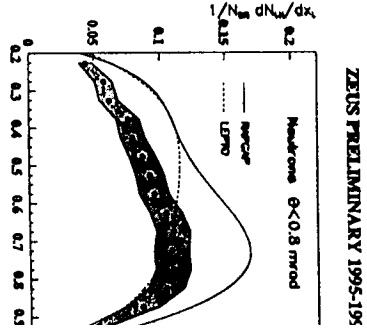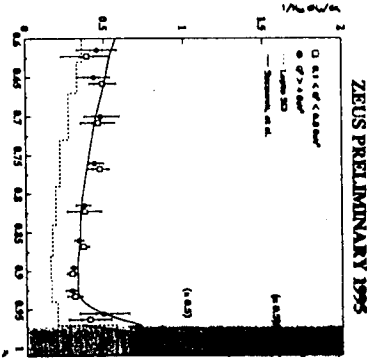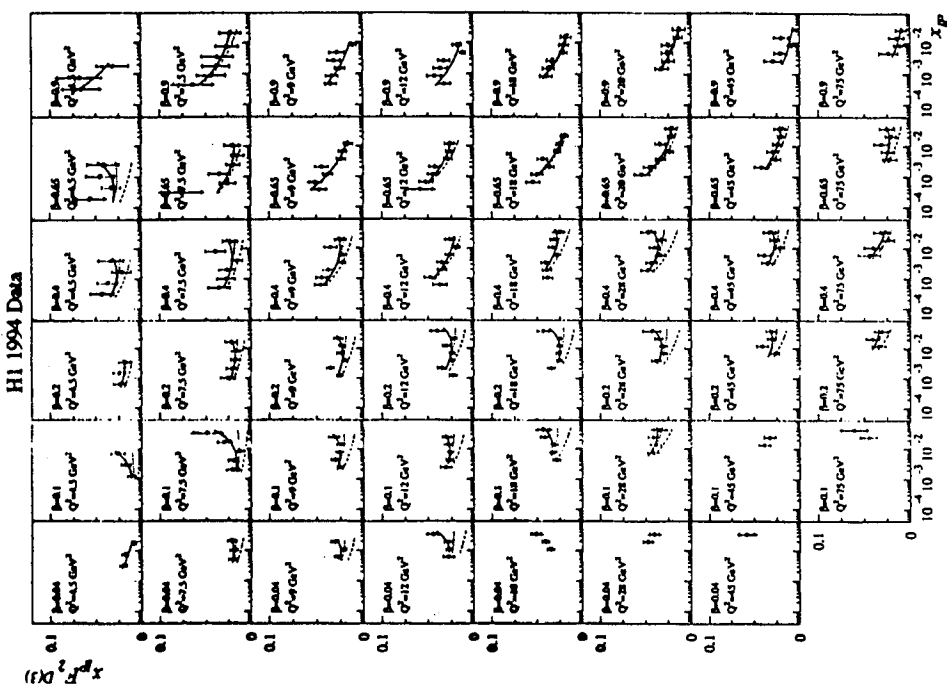
- Pomeron and Regge terms
- Small x
- α_P is 1.14 to 1.20. Soft pomeron 1.08–1.10
- Measurements also from ZEUS

30

ϕ y

η_{max} detector edge

X

] low mass /] mass

Deduce x_P from P_x^r/p^t
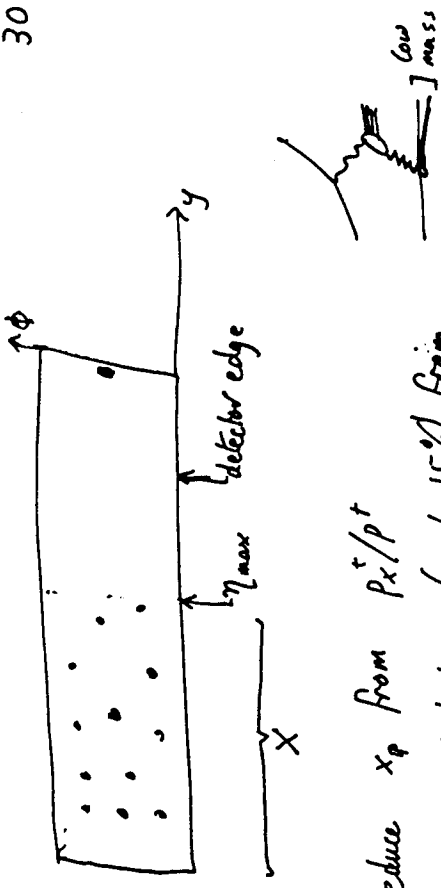
Some contribution (up to 15%) from

Much more data ~ 10% of all DIS at HERA

Analyses by ZEUS — m_x d/n

⊕ H1 — rap gap

OS:

$W=60\text{-}74\,GeV$ $W=110\text{-}134\,GeV$ $W=164\text{-}200\,GeV$

$Q^2=8\,GeV$ $Q^2=17\,GeV$

Events

$\ln(M_x^2)$
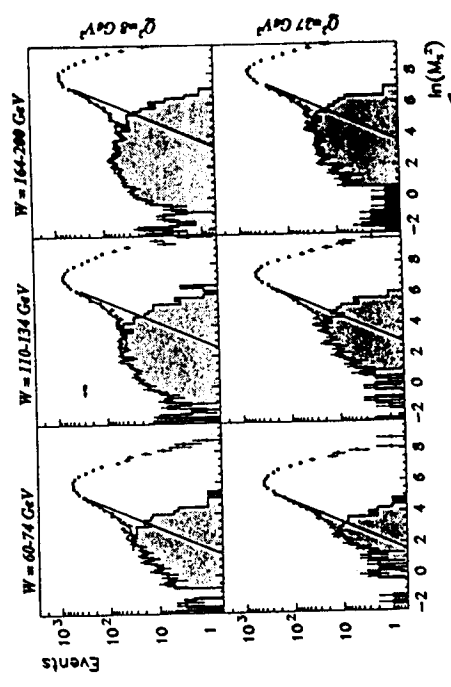
$L = \ln x_P + const.$

$$\text{quark}: \quad u = \bar{u} = d = \bar{d}$$

$$\left\{ \begin{array}{l} \text{gluon} \\ \text{Pomeron's } \alpha_P \end{array} \right.$$

1. H1   DIS. Include Reggeon term
2. ACTW: DIS & jet photoproduction. (H1 & ZEUS)
3. ZEUS: DIS & jet photoproduction

Dominant: DIS:
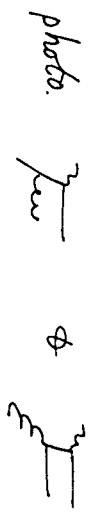
photo.

Large gluon component

⟹ Predictions (diffractive)

1. Scaling violations
2. DIS dijets
3. charm production ⟶ quark term NLO

• Photoproduction of jets:

$x_{I\!P} \, F_2^{D(3)}(x_{I\!P}=0.003)$

(a) Quarks Only (Fit 1)

$\beta=0.04$ $\beta=0.1$ $\beta=0.2$ $\beta=0.4$ $\beta=0.65$ $\beta=0.9$

Q^2 / GeV^2

$x_{I\!P} \, F_2^{D(3)}(x_{I\!P}=0.003)$

(b) Quarks + Gluons (Fit 3)

$\beta=0.04$ $\beta=0.1$ $\beta=0.2$ $\beta=0.4$ $\beta=0.65$ $\beta=0.9$

Q^2 / GeV^2

$d\sigma/d\eta_{jet}$ (pb)

$d\sigma/d\beta^{OBS}$ [pb]

ZEUS 1994

— hard quark + hard gluon
-- hard quark + leading gluon
··· hard quark + (hard + leading) gluon

β^{OBS}

● Gluon fraction:

	Prediction	
	Low glue	High glue

● Charm production:

Expt	Data	Low glue	High glue
H1:	$154 \pm 40 \pm 35$ pb	5-6	100-600
ZEUS:	$526 \pm 110 \pm \sim 220$	~ 3	100-500

● Diffractive fraction of charm production:

ZEUS 1995 - 1997 Preliminary

$\propto \beta(1-\beta) + \cdots$

$\propto \beta(1-\beta)$ or peaked at large β

$Q^2 = Q_0^2 = 4 \, GeV^2$ usually

		DIS size	DIS Q-dep	photo
glue	ACTW A&C H1 fit 1	✓	✗	✗
ne	ACTW B&D H1 fit 2 ZEUS hard	✓	✓	✓
peaked large β	ACTW SG H1 fit 3 ZEUS leading glue	✓	✓	✓

dσ/dz$_{IP}^{jets}$ [pb]

● H1 Data
— RAPGAP P.P.fit 2
— RAPGAP P.P.fit 3
···· LEPTO 6.5
⊠ qq̄ Fock State (2D)

10^2
10
1
10^{-1}

0.2   0.4   0.6   0.8   1

z$_{IP}^{jets}$
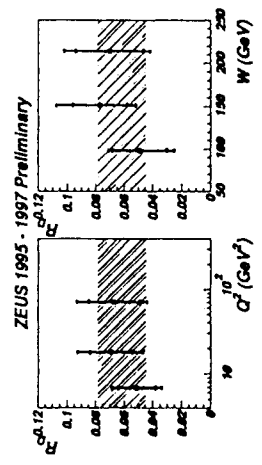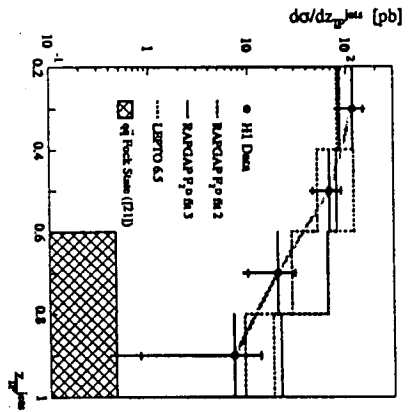
Is Ingelman-Schlein model correct QC

Yes, for hard scattering.
No, for Regge part.

Pomeron is **not** an off-shell particle

Not:

p

e'
e

p'

'current' quark jet

But:

p

e

e'

p'

→ time

Diffractive parton densities

(or (extended) fracture functions)

$$f^{(D)}(\beta,\mu;x_P,t) = \text{Fourier transform of}$$

$$\sum_X \langle P| \bar{\psi}(0,y^-,0_T)\, e^{iy^- A} |P'X\rangle\langle P'X|e^{-iy^- A}\,\psi(0)|P\rangle$$

Usual defn.: pdf = number density
but with restriction on final state.

Trentadue & Veneziano: fracture function

Universal — measure & use to predict other processes

DGLAP evolution — as usual.
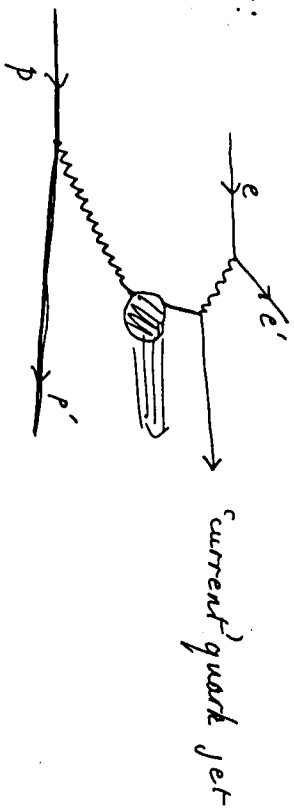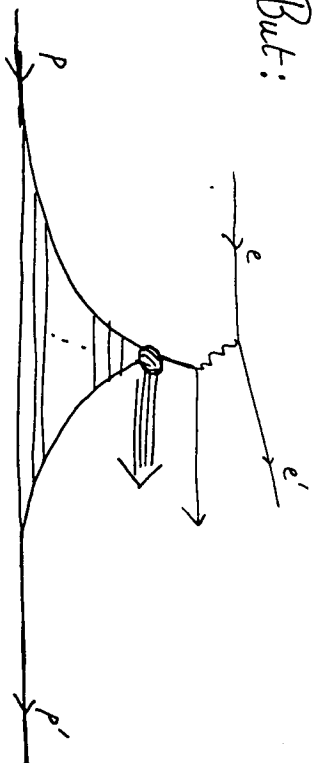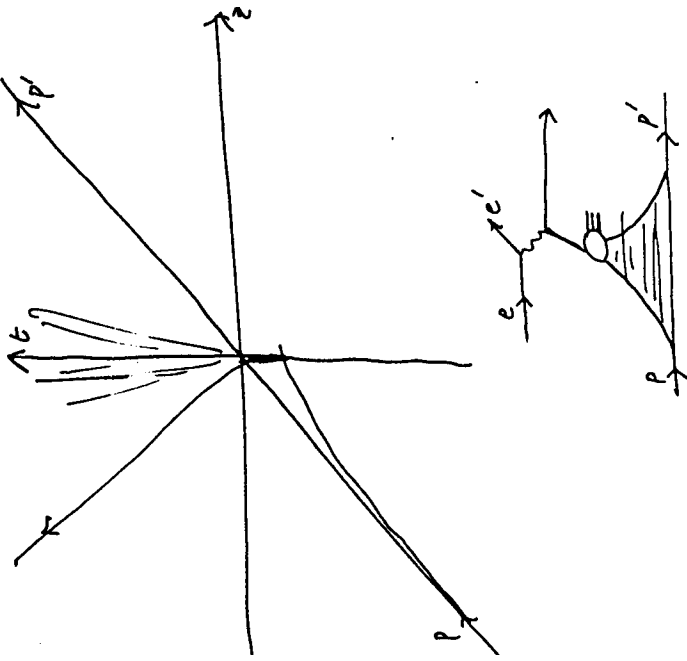
: • Prof of hard-scattering factorization (SCC)

Outgoing quark not affected by proton f.s.c
or effect is universal — in pdfs.

• Extends to general leptoproduction process

• NO proof of Regge factorization.

→ • Doesn't extend to hadron-hadron case.

$\gamma^*(q)$ slower   fast
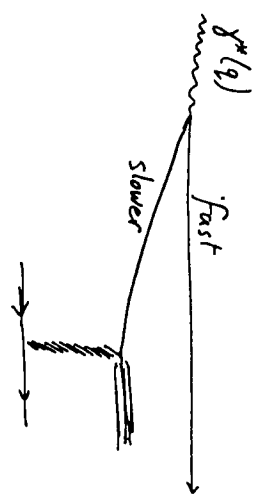
Aligned jet model. Bjorken

Relate physics to normal soft diffraction.

Models — diffraction v. ordinary small x
is a matter of different color
rearrangement of c

(Buchmüller & Hebecker, ...)

Valid model must be consistent with hard-
scattering factorization, especially for

$\frac{q}{\bar{q}}$

g
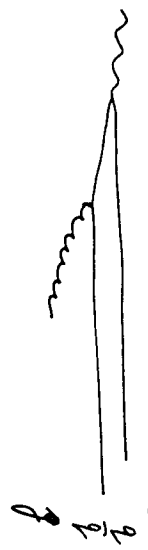
Hadron-hadron scattering
is different

p $\bar{p}$

← high p_T jets

$\bar{p}'$

Jets
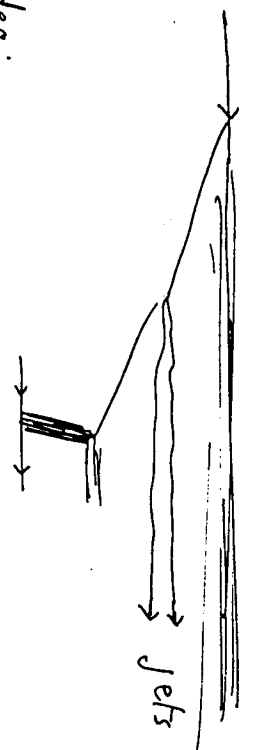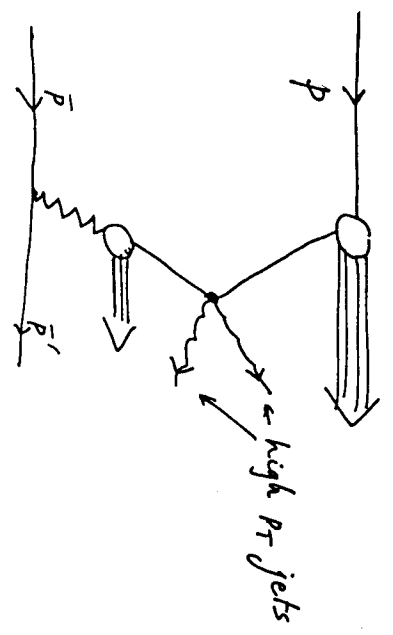
Idea:

Probability of diffraction
= $\left(\begin{array}{c}\text{probability for struck parton to leave} \\ p \text{ intact}\end{array}\right)$
× (prob. for same for rest of probe)

→ suppression.

Plot axes: $F_D^3(\beta)$ versus β

Legend:
- — H1 fit-2
- — H1 fit-3
- ($Q^2 = 75$ GeV^2)
- + CDF data
- $E_T^{jet1,2} > 7$ GeV
- $0.035 < \xi < 0.095$
- $|t| < 1.0$ GeV^2

N.B.: $\xi = x_P$, $\beta =$ parton momentum in Pomeron.

Extra interaction, "Pomeron" again break factorization, unless cross-section is completely inclusive.

un before QCD — see

De Tar, Ellis & Landshoff

1. Rapidity gap between high p_T jets
(in hadron-hadron collisions).

Basic model:

Jet 1
Jet 2
Rapidity gap

$$g + g \longrightarrow J_1 + J_2 \quad \text{at large } \hat{s}, \quad \hat{s} \gg |\hat{t}|.$$
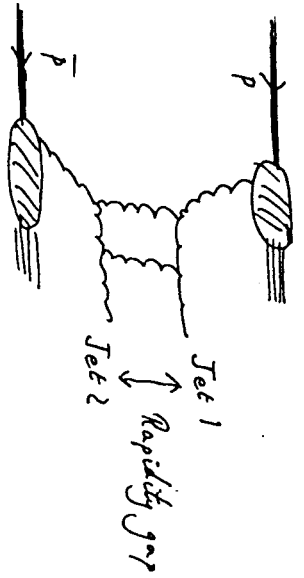
Pomeron & Regge physics for gluons!

$$\sigma = pdfs \times \hat{\sigma}(\text{color-singlet exchange}) \quad \alpha_s^2 \text{ suppression}$$
$$\times \text{ "gap survival probability"} \quad \sim 10\%$$

(Bjorken)

---

2. Exclusive DIS

e
e'
large Q
meson $(\rho, \phi \cdots)$ or ph
$\sum_n$
π
D
p
p'
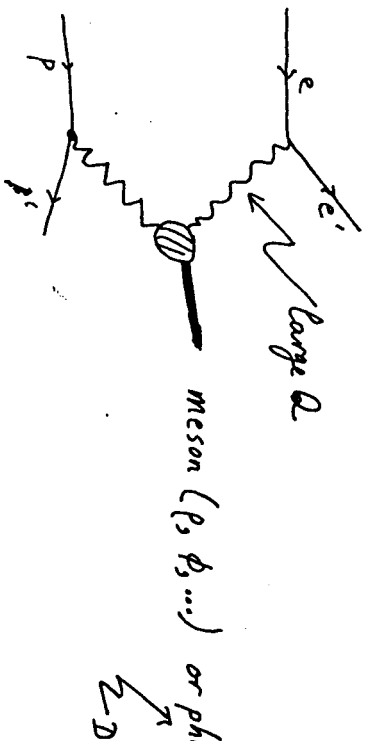
__Amplitude__ in terms of "__skewed pdfs__"

hard scattering
p
p'
$e \cdots$
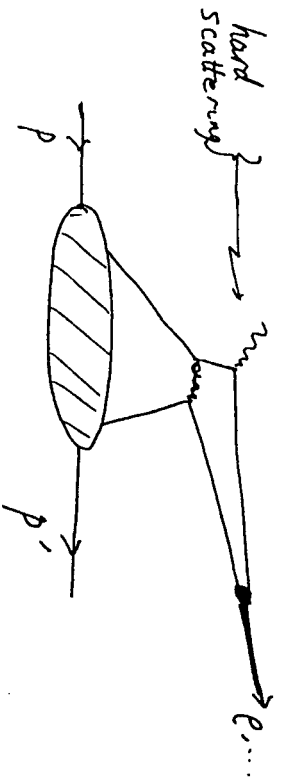
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1. Event structure in soft hadronic interactions
2. Light-cone coordinates, rapidity, pseudo-rapidity
3. Regge phenomenology; Pomeron
4. Space-time structure of scattering
5. Optical analogy
6. Basic diffraction
7. Diffractive hard scattering
   — DIS, jets, photoproduction of jets
     heavy quark production &c.
8. Diffractive pdfs; fracture functions
9. Modelling
10. Factorization breaking: eH v. HH
11. Exclusive DIS: $\gamma^* p \rightarrow \gamma p$ &c.
12. Skewed pdfs & their uses

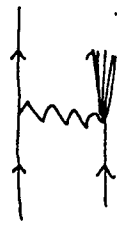Diffractive scattering: What kinds?
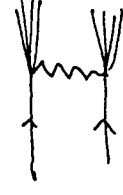
Elastic scattering at high energy & small angle
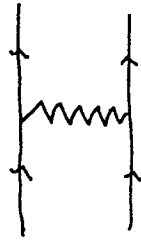
Single diffractive excitation (single diffraction') (SDE)

$$p + p \rightarrow p + X$$

forward isolated p; $M_x \ll \sqrt{s}$.

Double diffraction

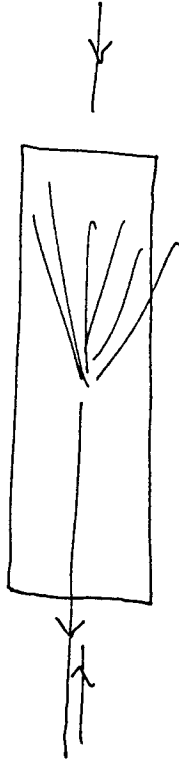2 isolated / low mass systems, relatively

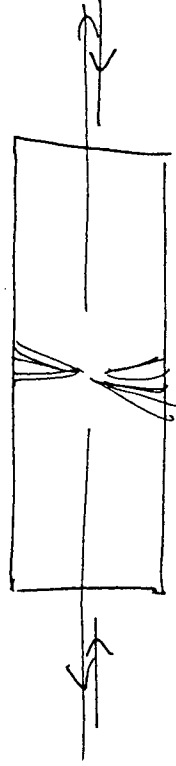Double pomeron exchange (DPE)

$$p + p \rightarrow p + X + p$$

central $M_x \ll \sqrt{s}$

SDE in collider detector:

DPE:

Light-cone Variables, Rapidity and All That

John C. Collins

Penn State University, 104 Davey Lab, University Park PA 16802, U.S.A.

(22 May 1997)

Abstract

I give a pedagogical summary of the methods of light-cone variables, rapidity and pseudo-rapidity. Then I show how these methods are useful in analyzing diffractive high-energy collisions.

I. INTRODUCTION

The use of light-cone variables, rapidity and pseudo-rapidity is very common in treating high-energy scattering, particularly in hadron-hadron and lepton-hadron collisions. The essential features of these collisions that make these variables of utility are the presence of ultra-relativistic particles and a preferred axis.

In these notes I will explain the theory of these variables. The only prerequisites are a knowledge of the elements of special relativity and an acquaintance with the general phenomenological features of high-energy scattering. Then I will show how these variables can be used conveniently in the analysis of the kinematics of diffractive processes. This will include the derivation of an approximate relation between the size of the rapidity gap and the variable x_P.

The material in these notes is essentially all standard and well known to many workers in the field. But it is not always easy to find the material conveniently presented.

II. LIGHT-CONE COORDINATES, RAPIDITY

A. Definitions

Light-cone coordinates are defined by a change of variables from the usual (t, x, y, z) (or $(0, 1, 2, 3)$) coordinates. Given a vector V^μ, its light-cone components are defined by

$$V^+ = \frac{V^0 + V^3}{\sqrt{2}}, \quad V^- = \frac{V^0 - V^3}{\sqrt{2}}, \quad V^T = (V^1, V^2), \tag{1}$$

and I will write $V^\mu = (V^+, V^-, V^T)$. Some authors prefer to omit the $1/\sqrt{2}$ factor in Eq. (1). It can easily be verified that Lorentz invariant scalar products have the form[1]

$$V \cdot W = V^+ W^- + V^- W^+ - V_T \cdot W_T,$$
$$V \cdot V = 2V^+ V^- - V_T^2. \tag{2}$$

What are the motivations for defining such coordinates, which evidently depend on a particular choice of the z axis? One is that these coordinates transform very simply under boosts along the z-axis. Another is that when a vector is highly boosted along the z axis light-cone coordinates nicely show what are the large and small components of momentum. Typically one uses light-cone coordinates in a situation like high-energy hadron scattering. In that situation, there is a natural choice of an axis, the collision axis, and one frequently needs to transform between different frames related by boosts along the axis. Commonly used frames include the rest frame of one of the incoming particles, the overall center-of-mass frame, and the center-of-mass of a partonic subprocess.

B. Boosts

Let us boost the coordinates in the z direction to make a new vector V'^μ. In the ordinary (t, x, y, z) components we have the well known formulae

$$V'^0 = \frac{V^0 + vV^z}{\sqrt{1-v^2}}, \quad V'^z = \frac{vV^0 + V^z}{\sqrt{1-v^2}}, \quad V'^x = V^x, \quad V'^y = V^y.$$

It is easy to derive the following for the light-cone components:

$$V'^+ = V^+ e^\psi, \quad V'^- = V^- e^{-\psi}, \quad V'^T = V_T, \tag{}$$

where the hyperbolic angle ψ is $\frac{1}{2}\ln\frac{1+v}{1-v}$, so that $v = \tanh\psi$.

Notice that if we apply two boosts of parameters ψ_1 and ψ_2, the result is a boost $\psi_1 + \psi_2$. This is clearly simpler than the corresponding result expressed in terms of velocities.

C. Rapidity

a. Boost of particle momentum Consider a particle of mass m that is obtained by boost ψ from the rest frame. Its momentum is

$$p^\mu = \left(p^+, \frac{m^2}{2p^+}, 0_T\right)$$
$$= \left(\frac{m}{\sqrt{2}}e^\psi, \frac{m}{\sqrt{2}}e^{-\psi}, 0_T\right)$$

---

[1] I prefer to make a distinction between contravariant vectors, whose indices are superscripts, and covariant vectors, whose indices are subscripts. In all contexts where the distinction matters I will put the Lorentz indices in their correct places.

e that if the boost is very large (positive or negative), only one of the two non-zero cone components of p^μ is large; the other component becomes small. With the usual onents two of the components, p^0 and p^z, become large.

uppose next that we have two such particles, p_1 and p_2, with the boost for particle 1 much larger than that for particle 2. Then in the scalar product of the two momenta one component of each momentum dominates the result, thus for example $(p_1+p_2)^2 \simeq$. This implies that, when analyzing the sizes of scalar products of highly boosted cles, it is simpler to use light-cone components than to use conventional components.

Definition of rapidity Since the ratio p^+/p^- gives a measure e^{2y} of the boost from st frame, we are led to the following definition of a quantity called "rapidity":

$$y = \frac{1}{2}\ln\frac{p^+}{p^-} = \frac{1}{2}\ln\frac{E+p^z}{E-p^z}, \quad (6)$$

n can be applied to a particle of non-zero transverse momentum. The 4-momentum of ticle of rapidity y and transverse momentum p_T is

$$p^\mu = \left(\sqrt{\frac{m^2+p_T^2}{2}}\,e^y, \sqrt{\frac{m^2+p_T^2}{2}}\,e^{-y}, p_T\right), \quad (7)$$

$\sqrt{m^2+p_T^2}$ being called the transverse energy E_T of the particle. It can be checked the scalar product of two momenta

$$p_1 \cdot p_2 = E_{1T}E_{2T}\cosh(y_1-y_2) - p_{1T}\cdot p_{2T}. \quad (8)$$

he case where the transverse momenta are negligible, this reduces to $m^2\cosh(y_1-y_2)$, ch is like the formula for the product of two Euclidean vectors $p_1 \cdot p_2 = p_1 p_2 \cos\theta$, with trigonometric cosine being replaced by the hyperbolic cosine.

c. Transformation under boosts Under a boost in the z direction, rapidity transforms tively:

$$y \to y' = y + \psi. \quad (9)$$

s implies that in situations where we have a frequent need to work with boosts along the xis it is economical to label the momentum of a particle by its rapidity and transverse mentum, rather than to use 3-momentum.

d. Rapidity distributions in high-energy collisions It also happens that in most collisions igh-energy hadronic scattering, the distribution of final-state hadrons is approximately form in rapidity, within kinematic limits. That is, the distribution of final-state hadrons pproximately invariant under boosts in the z direction. This implies that rapidity and nsverse momentum are appropriate variables for analyzing data and that detector ele nts should be approximately uniformly spaced in rapidity. (What is physically possible to make a detector uniform in the angular variable pseudo-rapidity that I will discuss ow.) This is in contrast to the situation for e^+e^- collisions where most of the interest in events generated via annihilation into an electro-weak boson. Such events are much ser to uniform in solid angle than uniform in rapidity.

e. Non-relativistic limit Observe that for a non-relativistic particle, rapidity is the same as velocity along the z-axis, for then

$$y = \frac{1}{2}\ln\frac{E+p^z}{E-p^z} \simeq \frac{1}{2}\ln\frac{m+mv^z}{m-mv^z} = v^z. \quad (10)$$

Non-relativistic velocities transform additively under boosts, and the non-linear change of variable from velocity to rapidity allows this additive rule Eq. (9) to apply to relativistic particles (but only in one direction of boost).

One way of seeing this is as follows. The relativistic law for addition of velocities in one dimension is

$$\beta_{13} = \frac{\beta_{12}+\beta_{23}}{1+\beta_{12}\beta_{23}}, \quad (11)$$

where β_{12} is the velocity of some object 1 measured in the rest-frame of object 2, etc. This formula is reminiscent of the following property of hyperbolic tangents:

$$\tanh(A+B) = \frac{\tanh A + \tanh B}{1+\tanh A\tanh B}. \quad (12)$$

So to obtain a linear addition law, we should write $\beta_{12} = \tanh A_{12}$, and then the rule Eq. (11) for the addition of velocities becomes simply $A_{13} = A_{12} + A_{23}$. The A variables are exactly relative rapidities, since

$$v^z = \frac{p^z}{E} = \frac{p^+ - p^-}{p^+ + p^-} = \tanh y. \quad (13)$$

f. Relative velocity Rapidity is the natural relativistic velocity variable. Suppose we have a proton and a pion with the same rapidity at $p_T = 0$. Then they have no relative velocity; to see this, one just boosts to the rest frame of one of the particles. But these same particles have very different energies: $E_p = \frac{m_p}{m_\pi}E_\pi$.

D. Pseudo-rapidity

As I will now explain, the rapidity of a particle can easily be measured in a situation where its mass is negligible, for then it is simply related to the polar angle of the particle. First let us define the pseudo-rapidity of a particle by

$$\eta = -\ln\tan\frac{\theta}{2}, \quad (14)$$

where θ is the angle of the 3-momentum of the particle relative to the $+z$ axis. It is easy to derive an expression for rapidity in terms of pseudo-rapidity and transverse momentum:

$$y = \ln\frac{\sqrt{m^2+p_T^2\cosh^2\eta}+p_T\sinh\eta}{\sqrt{m^2+p_T^2}}. \quad (15)$$

In the limit that $m \ll p_T$, $y \to \eta$. This accounts both for the name 'pseudo-rapidity' and for the ubiquitous use of pseudo-rapidity in high-transverse-momentum physics. Angles, and hence pseudo-rapidity, are easy to measure. But it is really the rapidity that is of physical significance: for example the distribution of particles in a minimum bias event is approximately uniform in rapidity over the kinematic range available.

The distinction between rapidity and pseudo-rapidity is very clear when one examines the kinematic limits on the two variables. In a collision of a given energy, there is a limit to the energy of the particles that can be produced. This can easily be translated to limits on the rapidities of the produced particles of a given mass. But there is no limit on the pseudo-rapidity, since a particle can be physically produced at zero angle (or at 180°), where its pseudo-rapidity is infinite. The particles for which the distinction is very significant are those for which the transverse momentum is substantially less than the mass. From Eq. (15) it follows that $y < \eta$ always.

III. DIFFRACTIVE SCATTERING

A. Kinematics

Let us define a single diffractive collision as one in which one of the beam particles emerges almost unscathed from the scattering, with only a small deflection and a small loss of energy (Fig. 1):

$$p + \bar{p} \to p' + X. \qquad (16)$$

For the sake of definiteness, I have assumed a proton-antiproton collision with the proton being diffracted. One can treat any other collision in the same fashion. Indeed, replacing the incoming and outgoing protons by electrons gives us electroproduction processes, and the low Q^2 events that are used to measure photoproduction cross sections have exactly the same kinematic properties as diffractive hadron collisions.

Suppose the particle p is moving in the $+z$ direction, and $\bar{p}$ in the opposite direction. Then we define the kinematics in terms of the transverse momentum p'_T of the diffracted hadron and a variable x_P defined as $1 - p'^+/p^+$. Thus the components of p and p' are

$$p^\mu = \left(p^+, \frac{m^2}{2p^+}, 0_T\right),$$

$$p'^\mu = \left((1 - x_P)p^+, \frac{m^2 + p'^2_T}{2(1-x_P)p^+}, p'_T\right). \qquad (17)$$

It is convenient to examine the momentum transfer:

$$\Delta^\mu \equiv p^\mu - p'^\mu = \left(x_P p^+, -\frac{x_P m^2 + p'^2_T}{2(1-x_P)p^+}, -p'_T\right). \qquad (18)$$

Clearly, x_P represents a fractional loss of the $+$ component of momentum. When the two protons are both moving fast, this fraction is also the fractional energy loss: $x_P = 1 - E'/E + O(1/E)$; but the definition in light cone components converts this to a boost-invariant definition. The invariant momentum transfer is
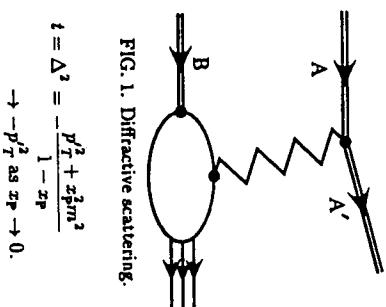
Thus at small x_P, t is an equivalent variable to the transverse momentum of the outgoing diffracted hadron.

FIG. 1. Diffractive scattering.

$$t = \Delta^2 = -\frac{p'^2_T + x_P^2 m^2}{1 - x_P}$$
$$\to -p'^2_T \text{ as } x_P \to 0. \qquad (19)$$

The diffractive region is when x_P is small enough that the power law attributed to pomeron exchange dominates. The exact nature of the pomeron in QCD is still quit... uncertain. But from a practical point-of-view one can identify it with whatever is exchange... between hadrons to make the approximately constant total cross sections that are observe... at high-energy. The object exchanged between the protons in Fig. 1 and the system labele... X is a pomeron when x_P is sufficiently small. In the Regge phenomenology of diffractiv... scattering, this is associated with an x_P distribution of the events that is approximatel... dx_P/x_P.

A more general definition of diffraction is that it consists of events in which there are no... exponentially suppressed rapidity gaps. The events that are diffractive in the first definitio... in fact are also diffractive according to the second definition. As I will show in Sect. III... events with small x_P have a rapidity gap that grows like $\ln(1/x_P)$. The power law associate... with pomeron exchange gives a distribution of events that is approximately dx_P/x_P, an... hence approximately a constant distribution in the size of the rapidity gap. That is, it giv... non-exponentially suppressed rapidity gaps.

In contrast, if one has a statistical distribution of particles, uniform in rapidity, and if a... the correlations between particles are local in rapidity, then the probability of a rapidity ga... of size Δy is proportional to $e^{-a\Delta y}$, for some constant a. Experimental data, for exampl... Fig. 2 indicate that for small gaps there is indeed an approximately exponential fall-off wi... increasing Δy. But for large enough gaps, more than 2 or 3 units, a presumably differe... mechanism of particle production becomes significant, and there is no longer a strong exp... nential drop. Of course, once Δy gets close to the kinematic limit, the distribution in Δ... must decrease again.

Strictly speaking, one should not talk about specific events as being diffractive or no... diffractive. Instead one should talk about the diffractive and non-diffractive components... a cross-section. When one says that a specific event is diffractive, one really means that it... in a kinematic region (of x_P or rapidity gap) where diffraction is the dominant contributi... to the cross section.
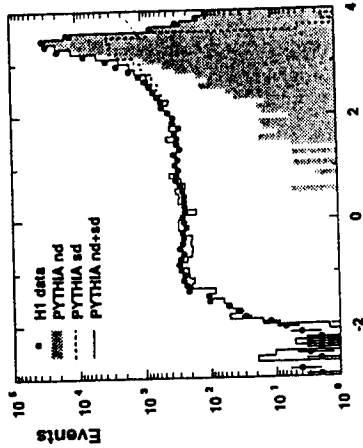
FIG. 2. Distribution of η_{max} in photoproduction at H1 [1]. There are no detected particles with pseudo-rapidity between η_{max} and the end of the detector at $\eta = 3.65$

B. Existence of a rapidity gap

Now the maximum value of the + component of the momentum of a hadron in the X of the final state in (16) is $\Delta^+ = z_P p^+$. It then follows from Eq. (7) that the maximum possible value of rapidity for any particle in X is

$$y_{min.\,max} = \ln \frac{2 z_P E}{m},$$ (20)

where E is the beam energy (assumed to be much larger than the proton mass). For a numerical example, consider a diffractive process with an incoming proton of energy GeV (as at the Fermilab Tevatron), and with $z_P = 0.01$, a value typically considered to safely in the diffractive region. The incoming proton itself has rapidity 7.6. From Eq. the maximum possible rapidity of pions in the event is 4.9. There is therefore a rapidity on the proton side of the event. Since there are normally many particles in the event, ich share the momentum, the edge of the rapidity gap, y_{max}, will typically be a unit or re lower.

The distribution of the soft particles in a hadronic collision is approximately uniform rapidity. So when the final-state particles are plotted in azimuth ϕ and rapidity y, a fractive event therefore looks like Fig. 3. When z_P is decreased, the value of $y_{min.\,max}$ also creases, and hence the size of the rapidity gap increases.

C. Computation of z_P from the non-diffracted part of the final state

In principle, z_P can be either measured from the final state hadron: $z_P = 1 - p'^+/p^+$ or om the rest of the final state:
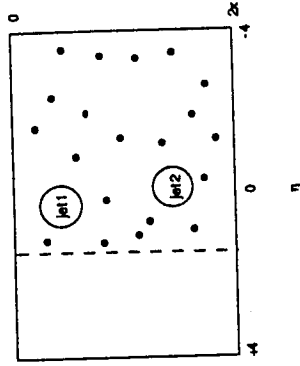
FIG. 3. Plot of the distribution of final-state particles in a typical diffractive event is uniform in ϕ and y except for an isolated forward proton.

$$z_P = \sum_{i \epsilon X} \frac{k_i^+}{p^+},$$ (21)

where the sum is over all particles in the remaining part X of the final state.

To measure z_P from the diffracted hadron requires first that there be elements of the detector (e.g., Roman pots) close to the beam-pipe, which is not always the case. Moreover, if z_P is very small, the hadron's energy needs to be known very precisely to obtain a useful accuracy on z_P; if the measurement is not sufficiently accurate, one cannot distinguish the value from zero. (Values of z_P in the range 10^{-4} to 10^{-2} are often discussed.) So if a useful measurement of z_P is not available, one must measure the rest of the hadronic final-state, and deduce z_P by using Eq. (21).

In principle, the use of Eq. (21) requires knowledge of all the (typically many) particles in X. One might imagine that poor information on X, for example from a lack of calorimetry in the backward part of the detector, would prevent such a measurement from being practical.

In fact, as I will now show, the value of the right-hand-side of Eq. (21) is dominated by the particles of the largest k_T (i.e., jet particles), and by the particles that are closest in rapidity to the diffracted hadron, i.e., closest to the edge of the rapidity gap.

We simply write each k_i^+ in Eq. (21) in terms of rapidity and transverse momentum to obtain

$$z_P = \sum_{i \epsilon X} \frac{E_{iT} e^{y_i}}{2E}.$$ (22)

Here I have approximated p^+ by $E\sqrt{2}$, with E being the energy of the incoming proton, which is assumed to be large. The transverse momentum appears in form of the transverse energy $E_{iT} = \sqrt{k_{iT}^2 + m^2}$. We next recall that particles in a hadronic scattering event are typically uniformly distributed in rapidity between some kinematic limits, and have a steeply falling spectrum in transverse momentum. The exponential of rapidity in Eq. (22) therefore implies that the sum is dominated by those particles of the highest rapidity.

The exception to this statement is given by particles that result from a hard scattering and that therefore have large transverse momentum. Hence the general statement is that the sum in Eq. (22) is dominated by (a) particles with the largest rapidity (i.e., closest to the edge of the rapidity gap), and (b) particles with high transverse momentum. This means that the measurement can be effectively be made from

$$x_P \simeq \sum_{\substack{\text{particles with highest } p_T \\ \text{particles with large } E_T}} \frac{E_{i,T}\,e^{y_i}}{2E}.$$ (23)

Hence a lack of detector coverage on the opposite side to the diffracted proton does not greatly affect the accuracy of the measurement.

Exactly this method is used in a corresponding problem in deep-inelastic scattering, where it is known as the Jacquet-Blondel [2] method. There it is desired to measure the scaling variable y (which has no relation to rapidity). This variable is defined in terms of the scattered lepton in exactly the way in which our x_P is defined in terms of the diffracted proton. By momentum conservation, y can be computed from the hadronic final state. The Jacquet-Blondel method is in regular use in deep-inelastic experiments at the HERA collider, and it provides useful accuracy. It is therefore evident that the same method applied to diffraction may well provide a practical method for measuring x_P.[2]

D. Scaling variable β_H relative to pomeron

When one has not only diffraction but also a hard scattering, there is another important kinematic variable that can be measured by a similar method. A closely related variable is the one called β in diffractive deep-inelastic scattering. I have added a subscript 'H' to distinguish my definition, with 'H' denoted 'hard scattering'. The definition and its physical interpretation can be explained by considering a diffractive hard collision treated by the QCD factorization formula, as formulated by Ingelman and Schlein [3]. The situation is represented in Fig. 4. There the hard part (i.e., the large p_T subprocess) of the collision is treated as being due to a collision of a parton from the antiproton with a parton out of the exchanged pomeron. The parton in the pomeron has a distribution that is a function of its momentum fraction, β_H, in the pomeron. Now fractions here are defined to be fractions of the + component of momenta. So we can measure β_H from the hadronic final state by

$$\beta_H = \frac{x_{jet}}{x_P},$$ (24)

where x_{jet} is defined in the same way as x_P, except that the sum is restricted to the particles that are generated by the hard scattering subprocess, i.e., the large transverse momentum particles. Thus

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[2]One counterargument is that when one measures y in deep-inelastic scattering, the particles dominating the sum corresponding to the one in Eq. (23) are those with large transverse momentum, whereas in diffractive hadron scattering, the soft particles, those in the underlying event, are important. In diffractive hadron scattering one will measure all the relevant low transverse momentum particles only if the tracking is very good.

9

$$\beta_H = \frac{\displaystyle\sum_{\text{particles with large } E_T} \frac{E_{i,T}\,e^{y_i}}{2E}}{\displaystyle\sum_{\text{all particles}} \frac{E_{i,T}\,e^{y_i}}{2E}}.$$ (25)

The sum in the denominator is effectively over just the particles with the largest rapidity and/or transverse energy. The importance of measuring cross sections differential in β_H is that it gives a direct probe of the parton densities in the pomeron.

I use the name 'jet' in the symbol x_{jet}, in view of the fact that an important application is to events containing high p_T jets. In that case the sums are actually over hadrons. However, it is important to observe that the same principles apply to any other hard process, like Drell-Yan. Then the sums are over all particles of the specified kinematics, hadronic or leptonic.

Important note about notation: In diffractive deep-inelastic scattering, it is common to define a variable $\beta \equiv x_{bj}/x_P$. In an event that is generated by the lowest-order mechanism of elastic electron-quark scattering, it can readily be verified that $\beta = \beta_H$. This is in fact a motivation for the definition of β.

Moreover, although the motivation for the definition of β_H, and many of its uses, are associated with the factorization formula (the 'Ingelman-Schlein model'), the reader should observe that the definition of β_H is completely independent of the model. The expression for β_H involves only properties of measurable hadrons (and leptons) in the collision. It does not involve the unobserved partons that are the mechanism by which the process is supposed to be generated. This observation is particularly important in the light of the fact that the factorization formula for diffractive hard scattering has not been proved from QCD, unlike the related formula for inclusive hard scattering.

E. Relation between rapidity gaps and x_P

Much diffractive data is presented as a function of the position of the edge of the pseudorapidity gap rather than as a function of x_P. So it is useful to obtain an approximate relation between the two variables, in order to be able to compare different sets of data. I will present results both for the gaps in rapidity and in pseudo-rapidity.

1. Estimation of the size of the gap

In a typical event, the distribution of particles as a function of y is supposed to look like Fig. 5(a). To estimate the relation between the edge of the rapidity gap and x_P, I approximate the true distribution by a theta function distribution, Fig. 5(b). In the approximate distribution we have a uniform distribution, for values of y less than a value I will call y_{edge}, and no particles above y_{edge}. The height of the distribution I will call dN/dy. From the distribution of hadrons in real diffractive events [4][3] I estimate that an appropriate value for dN/dy

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[3]There is also earlier data from the ISR [5].

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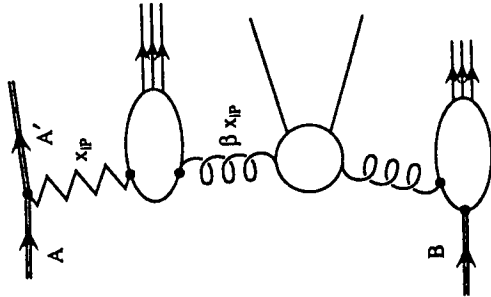
3. 4. Diffractive hard scattering. A parton inside the pomeron undergoes a scattering scattering with a parton out of the antiproton.

is about one unit less than the actual end of the rapidity gap, in order to obtain the value for z_P.

ext, I split the computation of z_P into a sum over the high p_T particles (e.g., associated jets), and a sum over the low p_T particles (which comprise what is often termed the rlying event'). For the particles in the underlying event, I replace their transverse y by its mean value $\langle E_T \rangle$. This gives

$$z_P \simeq \beta_H x_P + \int_{-\infty}^{y_{edge}} dy\, e^y \frac{dN}{dy} \frac{\langle E_T \rangle}{2E}$$
$$= \beta_H x_P + e^{y_{edge}} \frac{dN}{dy} \frac{\langle E_T \rangle}{2E}. \tag{26}$$

, I have extended the integral over y to $-\infty$, since, in the underlying event, only the ons closest to the edge of the rapidity gap are important in the calculation. Hence

$$\ln \frac{1}{x_P} \simeq -\ln \frac{1}{1-\beta_H} - y_{edge} - \ln \frac{dN}{dy} + \ln \frac{2E}{\langle E_T \rangle}$$
$$= -y_{max} - \ln\left(\frac{1}{1-\beta_H}\right) + \left[\ln \frac{2E}{\langle E_T \rangle} + y_{max} - y_{edge} - \ln \frac{dN}{dy}\right], \tag{27}$$

the term in square brackets being approximately independent of the event's kinematic ameters, y_{max} and β_H.
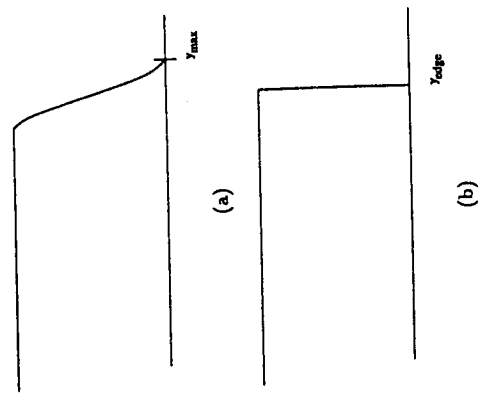

FIG. 5. (a) True distribution of particles in a diffractive event. (b) Theta-function approximation to this distribution.

2. Numerical values

For a relation between z_P and the maximum measured *pseudorapidity*, let us rewrite Eq. (27) as

$$\ln \frac{1}{x_P} = C - \eta_{max} - \ln \frac{1}{1-\beta_H} + \ln \frac{E}{900\,\text{GeV}}, \tag{28}$$

with

$$C = \eta_{max} - y_{edge} + \ln \frac{1800\,\text{GeV}}{\langle E_T \rangle} - \ln \frac{dN}{dy}. \tag{29}$$

Now C is a pure number that should be approximately independent of any of the parameters of the collision. So we can try to estimate it from the CDF data on single diffraction [4]. Their plots for the number distribution of *charged* tracks as a function of pseudo-rapidity are averaged over all events. Since different events have different values of x_P, the plot need not exhibit a rapidity plateau even if individual events do so. Moreover the rise of dN/dy from zero to its plateau will be spread out.

From the data, I estimate that at $\sqrt{s} = 1800$ GeV

$$\frac{dN}{d\eta} \approx 2,$$
$$\eta_{max} - y_{edge} \approx 1. \tag{30}$$

In addition I use the value [6] $\langle E_T \rangle \approx 0.3\,\mathrm{GeV}$. These values give $C = 8.5$.

Eq. (28) is, of course, not exact on an event-by-event basis. However, it should be useful on average, with the real data being smeared around the values related by the formula, and of course the numerical value '8.5' should be adjusted by a fit to ordinary single diffraction data.

There is indeed such data. CDF in Ref. [7] give a plot of η_{max} against x_P for their diffractive dijet data. Unfortunately, it disagrees quite badly with the above formula, both as to normalization and slope. This needs to be investigated. For example, the numbers I quoted above in Eq. (30) may be wrong, and my formulae have made no allowance for the fact that the measured η_{max} applies to particles above some value of $E_T \approx 400\,\mathrm{MeV}$. The true η_{max} may be rather larger.
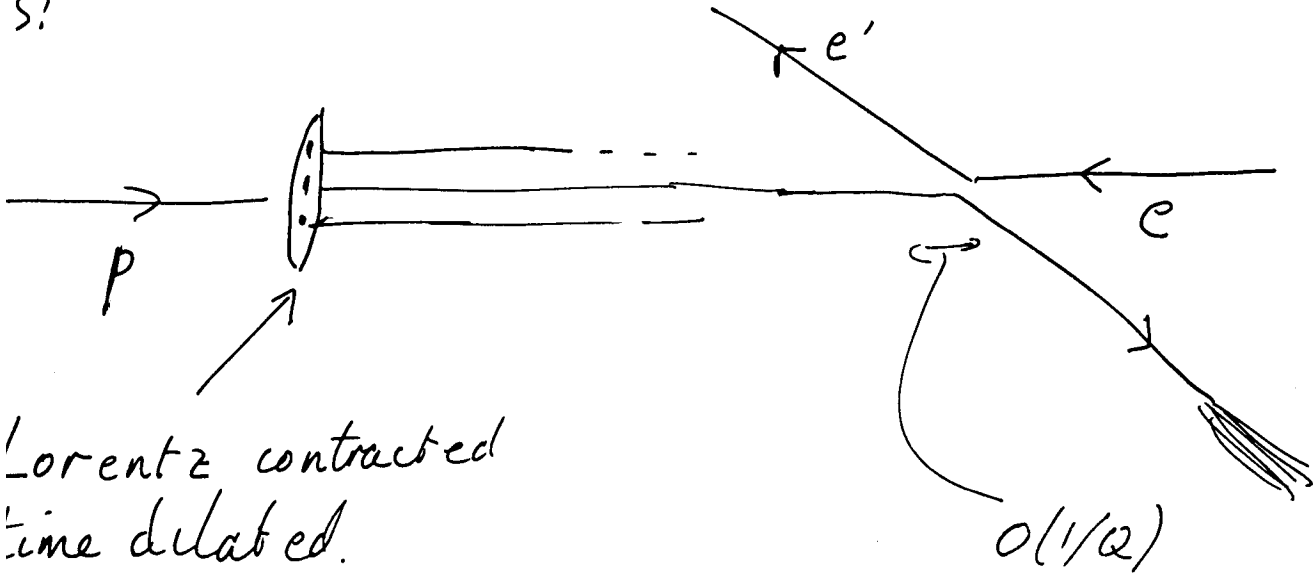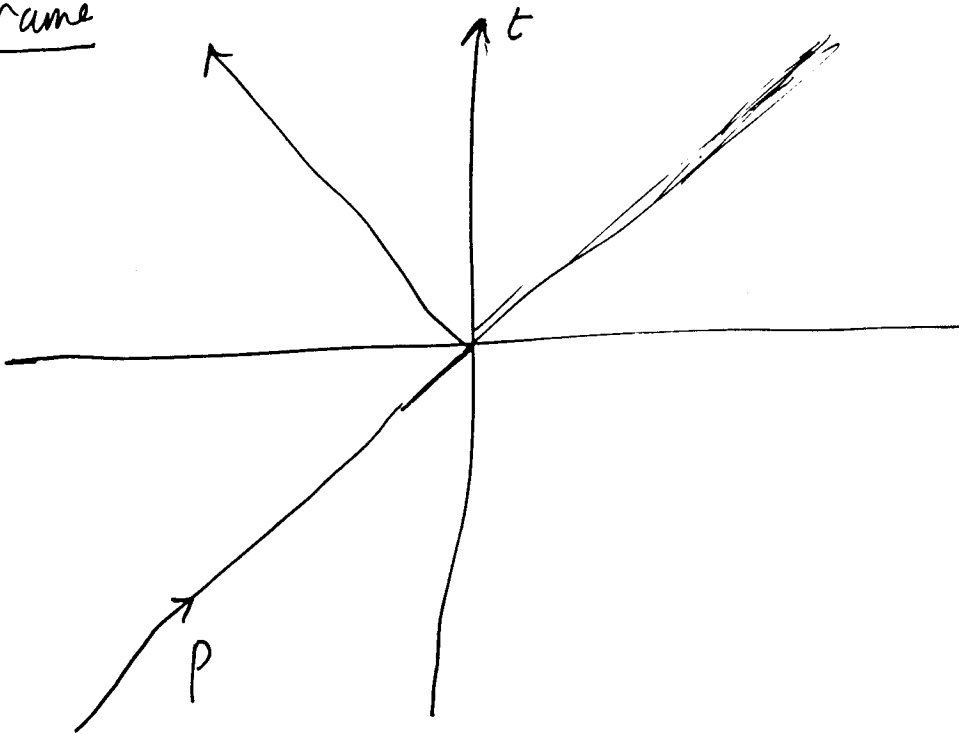
ACKNOWLEDGMENTS

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http://www-lib.fnal.gov/archive/1996/conf/Conf-96-231-E.ps.
In that paper, ξ is used to denote what I call x_P.

$$F_2 \qquad = \sum_a \int d\xi \qquad \hat{F}_2\left(\frac{x}{\xi}\right) \qquad f_{a/p}\left(\xi\right) + C.$$

S:

e'

e

P

Lorentz contracted
time dilated.

$O(1/Q)$

: frame

t

P

· approximately electron
scattering on free quarks &
gluons, moving in z-direction
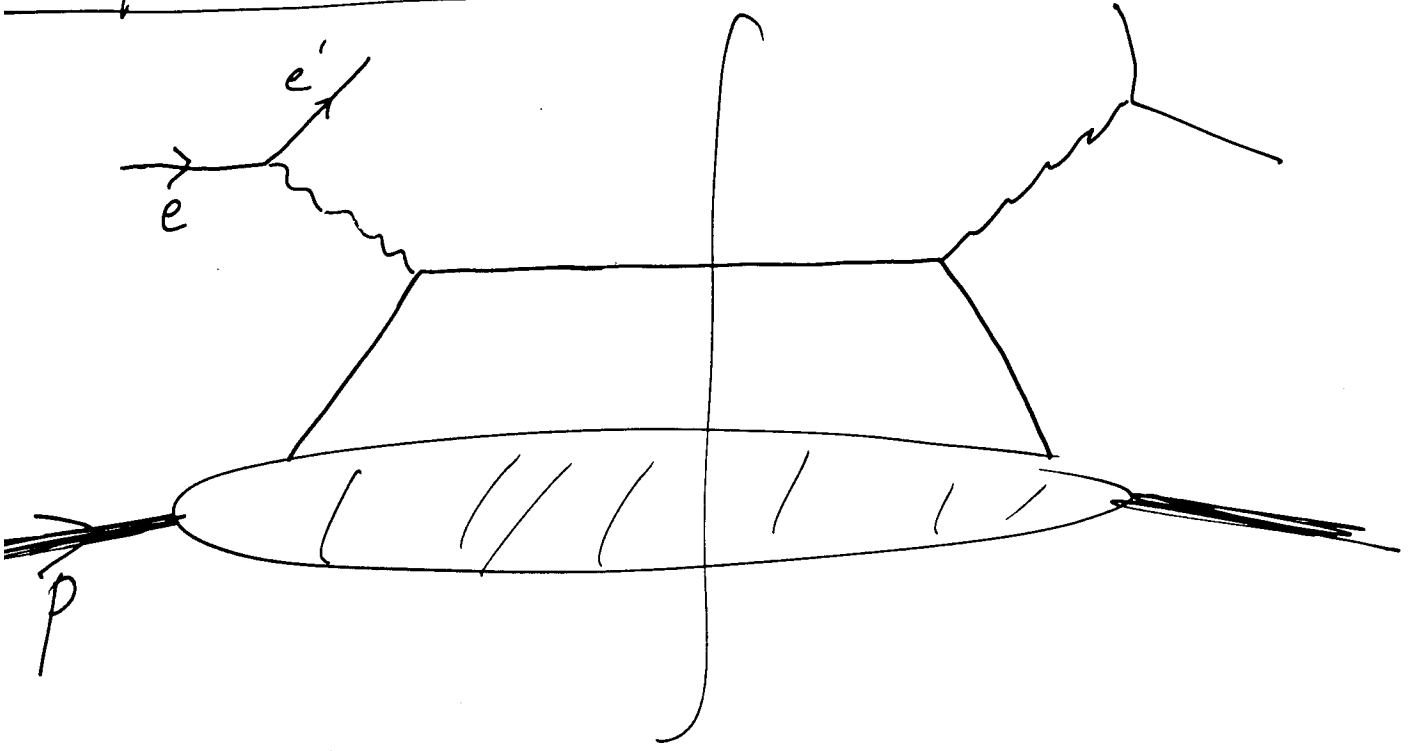
rs: $\sim m^2/Q^2$.

not $\qquad e + q + q$?
_____

Prob. of second parton

$\sim \dfrac{\text{Number of partons}}{\pi R^2} \times \pi\left(\dfrac{1}{Q}\right)^2$

$\sim \# \times \dfrac{1}{Q^2 R^2}$.

Implement in QFT:

Hadronization — too late to
ffect _inclusive_ cross-section
to correlate struck quark &
proton remnant

$\nearrow q \qquad k+q$

$>$

$k \nearrow$

$$p^\mu = \left(\underline{p}^+, \; p^- = small, \; 0_T\right)$$

$$q^\mu = \left(\underline{-xp^+}, \; \frac{Q^2}{2xp^+}, \; 0_T\right)$$

So $\quad \dfrac{Q^2}{2p \cdot q} = x$.

N.B. $\quad V^+ = \frac{1}{\sqrt{2}}\left(V^0 + V^z\right), \quad V^- = \frac{1}{\sqrt{2}}\left(V^0 - V^z\right)$

$$V^2 = 2V^+V^- - V_T^2.$$

$$' = (\xi p^+, small, small)$$

$$\simeq \int dk^+ dk^- d^2k_T \ Upper \times Lower$$

$$\simeq \int d\xi \, p^+ \ Upper\left(q, (\xi p^+, 0, 0_T)\right) \times$$

$$\times \left[\int dk^- d^2k_T \ Lower(p, k)\right]$$

$$= \int d\xi \, p^+ \ |amplitude|^2 \times \delta\left((\xi - x)\frac{Q^2}{x} - small\right)$$

$$\times pdf(\xi)$$

$$x \quad e_a^2 \ f_{a/p}(x)$$